Homework 4.

Deadline 5.1. 15:40

1. (4 points) Consider the variety of modules over a ring R. Prove that the free R-module $F(X)$ is isomorphic to $R^{(X)} = \{u \in R^X : \text{only finitely many coordinates } u_i \text{ are non-zero}\}$. Use the universal algebraic construction (via terms), do not use the categorical/module-theoretical definition of free-ness.

2. (8 points) Let $\mathbf{A} = (\{0,1\}, \cdot)$ where $x \cdot y = 0$ for all x, y. Let $\mathcal{V} = HSP(\mathbf{A})$. Determine a (small) equational basis of $\mathcal{V}$ (i.e., identities that axiomatize $\mathcal{V}$). Describe the free algebras in $\mathcal{V}$.

3. (8 points) Solve exercise 5 on p. 103 in Bergman's book.