Homework 3.

Deadline 15.12. 15:40

1. (6 points) Let $\mathcal{V}$ and $\mathcal{W}$ be varieties of groups and let $\mathcal{V} \circ \mathcal{W}$ denote the class of all groups G that possess a normal subgroup N such that $N \in \mathcal{V}$ and $G/N \in \mathcal{W}$. Show that $\mathcal{V} \circ \mathcal{W}$ is a variety.

2. (6 points) Let L, M be non-trivial lattices (i.e., more than one element). Define $L \oplus M$ to be the lattice with universe $L \cup M$ and ordered so that every element of L lies below every element of M. Prove that $HSP(\{L, M\}) = HSP(L \oplus M)$. (Hint: subdirect representation.)

3. (8 points) Prove that there is only one (up to isomorphism) subdirectly irreducible semilattice (i.e., commutative idempotent semigroup). (Hint: in idempotent algebras, congruence blocks are subalgebras. For the exercise, consider congruences with precisely one non-trivial block.)