Homework 2.

Deadline 24.11. 14:00

1. (6 points) Let $f, g : \mathbf{A} \to \mathbf{B}$ be homomorphisms.

(a) Prove that $\{a \in A : f(a) = g(b)\}$ is a subuniverse of $\mathbf{A}$. (As a past-time, you can also prove that this is an equalizer in the sense of category theory.)

(b) Assume that $\mathbf{A} = Sg(X)$. Prove that if $f|_X = g|_X$, then $f = g$.

(c) Prove that the number of homomorphisms $\mathbf{A} \to \mathbf{B}$ is at most $|B|^{|X|}$, where X is the smallest generating set of $\mathbf{A}$.

2. (8 points) Find all subalgebras and all congruences of $(\mathbb{N}, *)$ where $a*b = \max(a, b)+1$. Draw the lattices Sub, Con.

3. (6 points) Find all homomorphisms $(\mathbb{N}, \cdot)^2 \to (\{1, -1\}, \cdot)$. Here $\mathbb{N}$ does *not* contain zero.