Homework 1.

Deadline 1.11. 14:00

1. (8 points) Let $*$ be a quasigroup operation on a set Q and consider the corresponding division operations $/, \backslash$.

(a) Prove that if Q is finite, then $(Q, *)$ is term equivalent to $(Q, *, /, \backslash)$.
(b) Provide an example of an infinite Q such that $(Q, *)$ is term equivalent to $(Q, *, /, \backslash)$.
(c) Provide an example of an infinite Q such that $(Q, *)$ is *not* term equivalent to $(Q, *, /, \backslash)$.

(Examples require proofs!)

2. (6 points) Let $X = \mathbb{R}^n$ be a euclidean space. A subset $A \subseteq X$ is called convex if for every $a, b \in A$, the line segment between a, b belongs to A (i.e., if $a, b \in A$ then $ra + (1 - r)b \in A$ for every $r \in [0, 1]$). Prove that convex subsets form a complete lattice. What are its compact elements? Is it algebraic?

3. (6 points) Let C be a closure operator on a set X. Show that there is a Galois connection between X and some set Y such that C is the closure operator with respect to that connection.