Note: $|I| = |\Lambda| = 1 \Rightarrow M(G; \dots) \simeq G$ ... group ~~(group)~~

$|G| = 1,\ |\Lambda| = 1 \Rightarrow M(\dots)$ is left proj.

$|I| = 1$ right proj.

$|G| = 1 \Rightarrow$ it is in fact $I \times \Lambda$

$(i, \lambda) \circ (j, \mu) = (i, \mu)$

... diagonal semigroup or rectangular band $\}$ = left p. $\times$ right p.

(Fun fact: an idempotent groupoid is strongly abelian $\Leftrightarrow$ diagonal semigroup (rectangular band))

Rees' theorem (1940's) (special case of): S completely simple $\Leftrightarrow$ $S \simeq$ some Rees matrix smg

Proof: ($\Leftarrow$) easy to check

($\Rightarrow$) not at all easy $\boxtimes$

Coro (McLean's thm): Idempotent semigroups are semilattices of rectangular bands.

Proof: idemp. smg. $\Rightarrow$ comp. regular

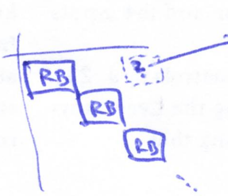
$\Rightarrow$ smlattice of Rees semigroups

$(i, a, \lambda) = (i, a, \lambda) \cdot (i, a, \lambda) = (i, a p_{\lambda i} a, \lambda) \quad \forall i, \lambda, a$

$\Rightarrow \cancel{a} = \cancel{a} p_{\lambda i} a$

$\Rightarrow 1 = p_{\lambda i} a \quad \forall a \Rightarrow |G| = 1$ $\square$

BUT: this is still rather weak:



?? (only know where to go) over semilattice

Petrich's theorem: an explicit construction of idempotent semigroups

(rect. bands, semilattice, some mappings $\rightsquigarrow$ id. smg.)