(1) $y(n+2) - y(n) = 17$

$y(1) = y(2) = 0$

- $\lambda^2 - 1 = 0$

$\lambda = \pm 1 \Rightarrow F.S. = \{1^n, (-1)^n\}$ (+2)

- $17 = 1^n (17 \cos(0n) + 0 \sin(0n))$

$\Rightarrow z(n) = n\, 1^n (a)$ pro nějaké $a \in \mathbb{R}$

$\Rightarrow a(n+2) - a \cdot n = 17$ $\Rightarrow z(n) = \frac{17}{2} n$ (+2)

$2a = 17$

$a = \frac{17}{2}$

obecné řešení: $y(n) = z(n) + c_1 1^n + c_2 (-1)^n$

$= \frac{17}{2} n + c_1 + c_2 (-1)^n, \quad c_1, c_2 \in \mathbb{R}$ (+2)

počáteční podmínky: $0 = y(1) = \frac{17}{2} + c_1 - c_2$

$0 = y(2) = 17 + c_1 + c_2$ (+3)

$\Rightarrow 0 = \frac{3}{2} \cdot 17 + 2c_1 \Rightarrow c_1 = -\frac{3}{4} \cdot 17$

$c_2 = \frac{17}{2} + c_1 = \frac{17}{2} - \frac{3}{4} \cdot 17 = -\frac{1}{4} \cdot 17$

řešení: $y(n) = \frac{17}{2} n - \frac{3}{4} 17 - \frac{1}{4} 17 (-1)^n, \quad n \in \mathbb{N}.$ (+1)

(2) $y' = \dfrac{1+y^2}{1+x^2}$, $y(0) = 1$

· $I = \mathbb{R}$, singulární řešení nejsou (+2)

· $J = \mathbb{R}$

· $\dfrac{y'}{1+y^2} = \dfrac{1}{1+x^2} \Rightarrow \operatorname{arctg} y = \operatorname{arctg} x + c$ (+2)

$y(0) = 1 \Rightarrow \operatorname{arctg} 1 = \operatorname{arctg} 0 + c$ (+2)

$\frac{\pi}{4} = c$

$$\underbrace{\operatorname{arctg} y(x)}_{G(y(x)),\ \text{kde } G(\mathbb{R}) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)} = \operatorname{arctg} x + \frac{\pi}{4}$$

$-\frac{\pi}{2} < \operatorname{arctg} x + \frac{\pi}{4} < \frac{\pi}{2}$ (+2)

$-\frac{3\pi}{4} < \operatorname{arctg} x < \frac{\pi}{4}$

$-\infty < x < 1 \Rightarrow y(x) = \operatorname{tg}\left(\operatorname{arctg} x + \frac{\pi}{4}\right)$, $x \in (-\infty, 1)$ (+2)

(3) $y' = \sqrt[3]{1-y^2}$ $I = \mathbb{R}$, singulární řešení $y = \pm 1$ (+2)

y	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
y'	$-$ $\searrow$	$+$ $\nearrow$	$-$ $\searrow$

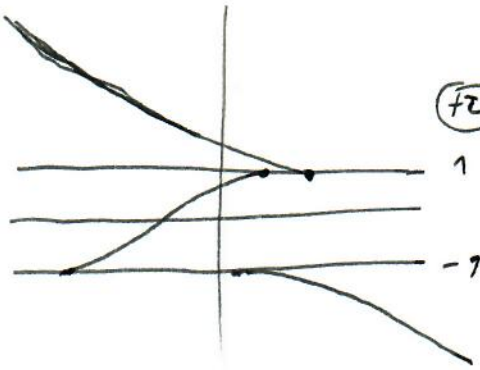
(+2)

u $-\infty$: srovnáme s $|y|^{2/3}$ ... diverguje (+1)

u -1: srovnáme s $|1+y|^{1/3}$ ... konverguje (+1)

u 1: srovnáme s $|1-y|^{1/3}$ ... konverguje (+1)

u ∞: srovnáme s $|y|^{2/3}$ ... diverguje (+1)



(+2)

4) $y'' - 2y' + 2y = \cos t$

$\lambda^2 - 2\lambda + 2 = 0$

$\lambda_{1,2} = \dfrac{2 \pm \sqrt{-4}}{2} = 1 \pm i \Rightarrow F.S. = \{e^t \cos t, e^t \sin t\}$ +2

$\cos t = e^{0\cdot t}(1 \cos(1\cdot t) + 0 \cdot \sin(1\cdot t)) \Rightarrow \mu + \nu i = 0 + 1\cdot i$

$\Rightarrow y(t) = t^0 e^{0\cdot t}(a \cos t + b \sin t) = a \cos t + b \sin t$ +2

$y' = -a \sin t + b \cos t$

$y'' = -a \cos t - b \sin t$

$\Rightarrow y'' - 2y' + 2y = -a \cos t - b \sin t + 2a \sin t - 2b \cos t + 2a \cos t + 2b \sin t$

$= \cos t(-a - 2b + 2a) + \sin t(-b + 2a + 2b)$ +2

$= \cos t\,(a - 2b) + \sin t\,(b + 2a)$

+2

$\Rightarrow$ $a - 2b = 1 \Rightarrow a + 4a = 1 \Rightarrow a = +1/5$

$b + 2a = 0 \Rightarrow b = -2a \qquad b = -2/5$

+1

$\Rightarrow y(t) = +\frac{1}{5} \cos t - \frac{2}{5} \sin t$

obecné řešení: $y(t) = +\frac{1}{5}\cos t - \frac{2}{5}\sin t + a e^t \cos t + b e^t \sin t$ +1

$t \in \mathbb{R},\ a, b \in \mathbb{R}$

5) $x' = x - 2y - z$

$y' = -x + y + z$

$z' = x - z$

$$\lambda I - A = \begin{pmatrix} \lambda - 1 & 2 & 1 \\ 1 & \lambda - 1 & -1 \\ -1 & 0 & \lambda + 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & \lambda - 1 & -1 \\ 0 & -(\lambda-1)(\lambda-1)+2 & (\lambda-1)+1 \\ 0 & -\lambda - 1 & \lambda \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda - 1 & -1 \\ 0 & -\lambda^2 + 2\lambda + 1 & \lambda \\ 0 & -\lambda - 1 & \lambda \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & \lambda-1 & -1 \\ 0 & -\lambda^2+\lambda+2 & 0 \\ 0 & \lambda-1 & \lambda \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda-1 & -1 \\ 0 & \lambda^2-\lambda-2 & 0 \\ 0 & \lambda-1 & \lambda \end{pmatrix}$$

+5

$$y'' - y' - 2y = 0$$

$$\lambda^2 - \lambda - 2 = 0 \Rightarrow \lambda_{1,2} = \frac{1}{2}\left(1 \pm \sqrt{1-4(-2)}\right) = \frac{1}{2}(1 \pm 3) = \begin{cases} 2 \\ -1 \end{cases}$$

- $y(t) = ae^{2t} + be^{-t}$
- $y' - y + z' = 0 \Rightarrow z' = y - y' = ae^{2t} + be^{-t} - 2ae^{2t} + be^{-t} = -ae^{2t} + 2be^{-t}$

$\Rightarrow z(t) = -\frac{a}{2}e^{2t} - 2be^{-t} + c$

- $x + y' - y - z = 0 \Rightarrow x(t) = z + y - y' = -\frac{a}{2}e^{2t} - 2be^{-t} + c - ae^{2t} + 2be^{-t}$

$= -\frac{3a}{2}e^{2t} + c \quad , \quad t \in \mathbb{R},\ a, b, c \in \mathbb{R}$

+5