1. $y(n+2) + 5y(n+1) - 6y(n) = n + 2^n$

$\lambda^2 + 5\lambda - 6 = (\lambda+6)(\lambda-1) \Rightarrow F.S. = \{(-6)^n, 1^n\}$ $+3$

• $q(n) = n \Rightarrow z(n) = n(an+b)$

$\Rightarrow n(14a) + (9a+7b) = n \Rightarrow a = \frac{1}{14}, \; b = -\frac{9a}{7} = -\frac{9}{98}$ $+3$

• $q(n) = 2^n \Rightarrow z(n) = a2^n \Rightarrow a = 1/8$ $+2$

$\Rightarrow y(n) = a(-6)^n + b1^n + \frac{1}{8}2^n + \left(\frac{1}{14}n - \frac{9}{98}\right)n, \; a, b \in \mathbb{R}, \; n \in \mathbb{N}$ $+2$

2. $y' = x^2\sqrt{y+2} \Rightarrow I = \mathbb{R}, \; J = (-2, \infty), \; y = -2$ sing. řešení $+2$

$$\frac{y'}{\sqrt{y+2}} = x^2$$

$$2\sqrt{y+2} = \frac{x^3}{3} + C, \quad y(0) = 0 \Rightarrow C = 2\sqrt{2}$$ $+2$ $+2$

$$\sqrt{y+2} = \frac{x^3}{6} + \sqrt{2} \Rightarrow \frac{x^3}{6} + \sqrt{2} > 0$$

$$x > -\sqrt[3]{3}\sqrt{2}$$ $+2$

$$y = \left(\frac{x^3}{6} + \sqrt{2}\right)^2 - 2, \quad x \in (-\sqrt[3]{3}\sqrt{2}, \infty)$$

lepení:
$$y = \begin{cases} \left(\frac{x^3}{6} + \sqrt{2}\right)^2 - 2, & x \in (-\sqrt[3]{3}\sqrt{2}, \infty) \\ -2, & x \in (-\infty, -\sqrt[3]{3}\sqrt{2}\rangle \end{cases}$$ $+2$

3. $y' = (y-1)^2 \log(1+y^2)$

$J = (-\infty, 0), (0,1), (1,\infty)$ +2

$g > 0$ na intervalu $J \Rightarrow y$ vrostoucí zde

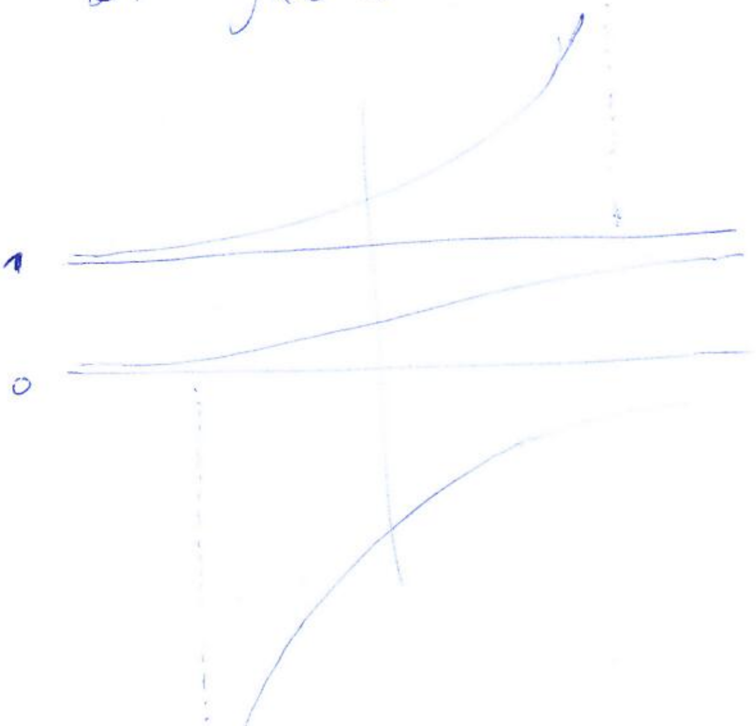
$\int \frac{1}{g} = \int \frac{1}{(y-1)^2 \log(1+y^2)}$ konvergence:

$-\infty$: $|\frac{1}{g}| \le \frac{1}{(y-1)^2}$ což srovnáme s $\frac{1}{y^2}$, což konverguje +1

0: srovnám s $\frac{1}{\log(1+y^2)}$, což srovnáme s $\frac{1}{y^2}$, což diverguje +2

1: srovnám s $\frac{1}{(y-1)^2}$, což diverguje +2

∞: jako u $-\infty$ +1



+2

4. $y''' + y = 0$

$\lambda^3 + 1 = 0$

$(\lambda+1)(\lambda^2 - \lambda + 1) = 0 \Rightarrow \lambda = -1, \frac{1}{2} + i\frac{\sqrt{3}}{2}, \frac{1}{2} - i\frac{\sqrt{3}}{2}$ +3

$\Rightarrow F.S. = \{ e^{-t}, e^{\frac{t}{2}} \cos \frac{\sqrt{3}}{2}t, e^{\frac{t}{2}} \sin \frac{\sqrt{3}}{2} t \}$ +2

$z(t) = a + bt + ct^2 \Rightarrow z'''(t) = 0 \Rightarrow c = 1$ +3

$\Rightarrow y(t) = a e^{-t} + b e^{t/2} \cos \frac{\sqrt{3}}{2} t + c e^{t/2} \sin \frac{\sqrt{3}}{2} t + t^2, \quad a, b, c \in \mathbb{R}, \; t \in \mathbb{R}$ +2

5. $\begin{pmatrix} \lambda - 6 & 7 & -4 \\ -1 & \lambda & -1 \\ 2 & -3 & \lambda \end{pmatrix}$ $\lambda - 6$ $\sim$ $+2$

$\sim \begin{pmatrix} 0 & \lambda^2 - 6\lambda + 7 & -\lambda + 2 \\ -1 & \lambda & -1 \\ 0 & 2\lambda - 3 & \lambda - 2 \end{pmatrix} + \sim \begin{pmatrix} 0 & \lambda^2 - 4\lambda + 4 & 0 \\ -1 & \lambda & -1 \\ 0 & 2\lambda - 3 & \lambda - 2 \end{pmatrix}$ +4

$\Rightarrow y'' - 4y' + 4y = 0 \qquad \Rightarrow y = a e^{2t} + b t e^{2t}$

$\lambda^2 - 4\lambda + 4 = (\lambda - 2)^2$

$\Rightarrow z' - 2z = 3y - 2y' = -e^{2t}(a + 2b) - b t e^{2t}$

$F.S. = \langle e^{2t} \rangle \qquad z(t) = c e^{2t} + (-a - 2b) t e^{2t} - \frac{b}{2} t^2 e^{2t}$ +6

$\Rightarrow x = y' - z = 2a e^{2t} + b e^{2t} + 2b t e^{2t} - z(t)$

$= e^{2t}(2a - c + b) + t e^{2t}(a + 4b) + t^2 e^{2t} \frac{b}{2}$