(1.) $y(n+2) + 10y(n+1) + 24y(n) = 3^n$

$\lambda^2 + 10\lambda + 24 = (\lambda+4)(\lambda+6) \Rightarrow F.S. = \{(-4)^n, (-6)^n\}$ +3

$z(n) = a3^n$, pak $a(3^{n+2} + 10\cdot 3^{n+1} + 24\cdot 3^n) = 3^n$

$a(9 + 30 + 24) = 1$ +5

$a = \frac{1}{63}$

$\Rightarrow y(n) = c_1(-4)^n + c_2(-6)^n + \frac{1}{63}3^n, \quad c_1, c_2 \in \mathbb{R}, n \in \mathbb{N}$ +2

(2.) $y' = \cot g\, x \; y^3, \quad y(\pi/4) = 1$

$I = (0,\pi) + k\pi, k \in \mathbb{Z}, \quad J = (-\infty, 0), (0,\infty), \quad y = 0$ je singulární řešení +2

vzhledem k podmínce máme $I = (0,\pi), \; J = (0,\infty)$

$\frac{y'}{y^3} = \cot g\, x \Rightarrow -\frac{1}{2}y^{-2} = \log|\sin x| + c$ +4 $\qquad y(\frac{\pi}{4}) = 1$

$-\frac{1}{2} = \log\frac{1}{\sqrt{2}} + c$

$= -\log\sqrt{2} + c$

$c = \log\sqrt{2} - 1/2$ +2

$-2c = -2\log\sqrt{2} + 1 = 1 - \log 2$

$\Rightarrow y = \frac{1}{\sqrt{1 - \log 2 - 2\log(\sin x)}}, \quad x \in (0,\pi)$ +2

(3.) $y' = \underbrace{\frac{\sqrt{|y-5|}}{\sqrt[3]{y}}}_{g(y)}$

g: $-$ 0 $+$ 5 $+$

singulární řešení je $y = 5$ +2

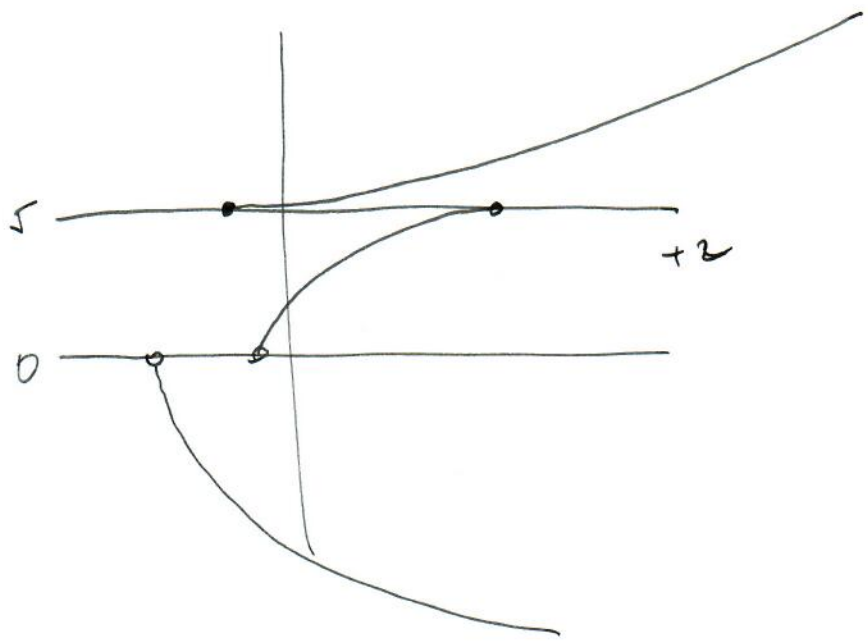
$\int \frac{1}{g(y)} = \int \frac{\sqrt[3]{y}}{\sqrt{|y-5|}}$

$-\infty$: srovnáme s $|y|^{1/3 - 1/2}$, což diverguje +2

0: konverguje +1

5: konverguje, ~~[illegible]~~ +1

∞: srovnáme s $y^{1/3-1/2}$, což diverguje +2



+2

④ $y'' + y = x + e^x$ +4

$\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i \Rightarrow F.S. = \{\cos x, \sin x\}$

$y_1 = a e^x \Rightarrow a = \frac{1}{2} \Rightarrow y_1 = \frac{1}{2} e^x$ +6

$y_2 = x$

$\Rightarrow y(x) = a \cos x + b \sin x + x + \frac{1}{2} e^x, \quad a, b \in \mathbb{R}, \ x \in \mathbb{R}$

⑤ $\begin{pmatrix} \lambda-7 & 10 & 4 \\ -4 & \lambda+7 & 4 \\ 6 & -7 & \lambda-1 \end{pmatrix} \begin{matrix} \uparrow \frac{1}{4}(\lambda-7) \\ \\ \downarrow \frac{1}{4}6 \end{matrix} \sim \begin{pmatrix} 0 & \frac{1}{4}(\lambda^2-49)+10 & \lambda-3 \\ -4 & \lambda+7 & 4 \\ 0 & \frac{3}{2}(\lambda+7)-7 & \lambda+5 \end{pmatrix} \ -$

$\sim \begin{pmatrix} 0 & \frac{1}{4}(\lambda^2-6\lambda-23) & -8 \\ -4 & \lambda+7 & 4 \\ 0 & \frac{3}{2}(\lambda+7)-7 & \lambda+5 \end{pmatrix} \sim \ \frac{1}{8}(\lambda+5)$

$\sim \begin{pmatrix} 0 & \frac{1}{4}(\lambda^2-6\lambda-23) & -8 \\ -4 & \lambda+7 & 4 \\ 0 & \lambda^3-\lambda^2-5\lambda-3 & 0 \end{pmatrix}$ +6

5. $\lambda^3 - \lambda^2 - 5\lambda - 3 = 0$

$(\lambda - 3)(\lambda + 1)^2 = 0 \Rightarrow y(t) = a e^{3t} + b e^{-t} + c t e^{-t}$

+4 $\quad z(t) = \frac{1}{32}(y'' - 6y' - 23y) = -a e^{3t} + e^{-t}\left(-\frac{c}{4} - \frac{b}{2}\right) + t e^{-t}\left(-\frac{c}{2}\right)$

$x(t) = \frac{1}{4}(y' + 7y) + z = \frac{3}{2} a e^{3t} + b e^{-t} + c t e^{-t}$