1. $y(n+2) - y(n+1) - 2y(n) = n$

$\lambda^2 - \lambda - 2 = (\lambda+1)(\lambda-2) \Rightarrow FS = \{(-1)^n, 2^n\}$ +3

$PS = 1^n(a\cos 0n + 0\sin 0n) \Rightarrow z(n) = a + bn$ +2

Pak $(a+b(n+2)) - (a+b(n+1)) - 2(a+bn) = -2a - 2bn + b = n$

$\Rightarrow -2b = 1 \qquad 0 = -2a + b$ +3

$b = -1/2 \qquad a = \frac{b}{2} = -1/4$

$\Rightarrow z(n) = -1/4 - 1/2\, n \Rightarrow y(n) = c_1(-1)^n + c_2 2^n + (-1/4 - 1/2\, n),\ c_1, c_2 \in \mathbb{R}$ +2

2. $y' = \cos x \sin y$, $y(0) = \frac{\pi}{2}$

$I = \mathbb{R}$, $J = (0,\pi) + k\pi$, zejména $I = \mathbb{R}$, $J = (0,\pi)$ +2

Pak $\frac{y'}{\sin y} = \cos x$, kde $\int \cos x = \sin x$

$\int \frac{1}{\sin y} = \int \frac{1}{2\sin\frac{y}{2}\cos\frac{y}{2}} = \int \frac{1}{\operatorname{tg}\frac{y}{2}\cos^2\frac{y}{2}} = \log|\operatorname{tg}\frac{y}{2}|$ +4

$y \in (0,\pi)$

$\Rightarrow \log|\operatorname{tg}\frac{y}{2}| = \sin x + c$ +2

$y(0) = \frac{\pi}{2} \Rightarrow 0 = \log|\operatorname{tg}\frac{\pi}{4}| = \log|\operatorname{tg}\frac{y(0)}{2}| = \sin 0 + c = c$

$\Rightarrow \log|\operatorname{tg}\frac{y}{2}| = \sin x$, $y \in (0,\pi) \Rightarrow \operatorname{tg}\frac{y}{2} > 0$

$\operatorname{tg}\frac{y}{2} = e^{\sin x}$

$y = 2\operatorname{arctg}(e^{\sin x})$, $x \in \mathbb{R}$ +2

3. $y' = (y^3 - 8)(y - \sin y)$

$g(y) = 0 \iff y = 2, y = 0$

$g(y) > 0$ na $(-\infty, 0) \cup (2, \infty)$ +1

$g(y) < 0$ na $(0, 2)$

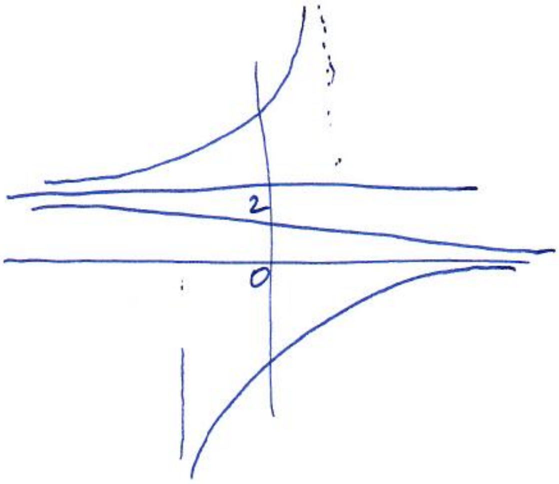
$\int \frac{1}{g(y)}$:

$-\infty$: srovnáme s $\frac{1}{y^4}$, což konverguje +2

0 : srovnáme s $\frac{1}{y^3}$, což diverguje +2

2 : srovnáme s $\frac{1}{y-2}$, což diverguje +2

∞ : srovnáme s $\frac{1}{y^4}$, což konverguje +2



+1

(4.) $y'' + 4y = \sin x$

$\lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i \Rightarrow FS = \{\sin 2x, \cos 2x\}$ +3

$y_p = a\cos x + b\sin x \Rightarrow y_p'' + 4y = \cos x(-a + 4a) + \sin x(-b + 4b) = \sin x$

$y_p' = -a\sin x + b\cos x$ $\quad 3a = 0 \Rightarrow a = 0$ +6

$\quad 3b = 1 \Rightarrow b = 1/3$

$y_p'' = -a\cos x - b\sin x$

$y = a\sin 2x + b\cos 2x + 1/3 \sin x\,, x \in \mathbb{R}\,, a, b \in \mathbb{R}$ +1

(5.) $\begin{pmatrix} \lambda - 1 & 6 & -3 \\ 0 & \lambda + 8 & -6 \\ -3 & 12 & \lambda - 7 \end{pmatrix} \frac{1}{3}(\lambda - 1) \sim \begin{pmatrix} 0 & 4\lambda + 2 & \frac{1}{3}(\lambda^2 - 8\lambda + 7) - 3 \\ 0 & \lambda + 8 & -6 \\ -3 & 12 & \lambda - 7 \end{pmatrix} \sim$

$\sim \begin{pmatrix} 0 & 12\lambda + 6 & \lambda^2 - 8\lambda - 2 \\ 0 & \lambda + 8 & -6 \\ -3 & 12 & \lambda - 7 \end{pmatrix} -12 \sim \begin{pmatrix} 0 & -90 & \lambda^2 - 8\lambda + 70 \\ 0 & \lambda + 8 & -6 \\ -3 & 12 & \lambda - 7 \end{pmatrix} \frac{1}{90}(\lambda + 8)$

$\sim \begin{pmatrix} 0 & -90 & \lambda^2 - 8\lambda + 70 \\ 0 & 0 & \lambda^3 + 6\lambda + 20 \\ -3 & 12 & \lambda - 7 \end{pmatrix}$ +6

5. $\lambda^3 + 6\lambda + 20 = (\lambda+2)(\lambda^2 - 2\lambda + 10)$

$\lambda_1 = -2 \ , \ \lambda_{2,3} = \frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm i6}{2} = 1 \pm i3$

$FS = \{ e^{-2t}, \ e^{t} \cos 3t, \ e^{t} \sin 3t \}$

$z(t) = a e^{-2t} + b e^{t} \cos 3t + c e^{t} \sin 3t$

$y(t) = \frac{1}{50}(z'' - 8z' + 70z) = a e^{-2t} + e^{t} \cos 3t \left(-\frac{1}{5}c + \frac{3}{5}b\right) + e^{t} \sin 3t \left(\frac{b}{5} + \frac{3}{5}c\right)$

$x(t) = +\frac{1}{3}\left(12y + z' - 7z\right) = 4y + \frac{1}{3}(z' - 7z) =$

$= a e^{-2t} + \left(\frac{2}{5}b + \frac{1}{5}c\right)(e^{t} \cos 3t) + \left(-\frac{1}{5}b + \frac{2}{5}c\right)(e^{t} \sin 3t)$

$a, b, c \in \mathbb{R}, \ t \in \mathbb{R}.$

+4