(5.) $x' = 5x + y - z$

$y' = x + 3y + z$

$z' = 7x + 3y + z$

$$\lambda I - A = \begin{pmatrix} \lambda-5 & -1 & 1 \\ -1 & \lambda-3 & -1 \\ -7 & -3 & \lambda-1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} -1 & \lambda-3 & -1 \\ \lambda-5 & -1 & 1 \\ -7 & -3 & \lambda-1 \end{pmatrix} \begin{matrix} \\ \cdot(\lambda-5) \\ \cdot(-7) \end{matrix} \sim \begin{pmatrix} -1 & \lambda-3 & -1 \\ 0 & (\lambda-3)(\lambda-5)-1 & -\lambda+5+1 \\ 0 & -7\lambda+18 & \lambda+6 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & \lambda-3 & -1 \\ 0 & \lambda^2-8\lambda+16 & -\lambda+6 \\ 0 & -7\lambda+18 & \lambda+6 \end{pmatrix} \downarrow \sim \begin{pmatrix} -1 & \lambda-3 & -1 \\ 0 & \lambda^2-8\lambda+16 & -\lambda+6 \\ 0 & \lambda^2-15\lambda+32 & 12 \end{pmatrix} \uparrow \cdot\frac{1}{12}(\lambda-6)$$

$$\sim \begin{pmatrix} -1 & \lambda-3 & -1 \\ 0 & \frac{1}{12}(\lambda^3-9\lambda^2+26\lambda-24) & 0 \\ 0 & \lambda^2-15\lambda+32 & 12 \end{pmatrix} \sim \begin{pmatrix} -1 & \lambda-3 & -1 \\ 0 & (\lambda-2)(\lambda-3)(\lambda-4) & 0 \\ 0 & \lambda^2-15\lambda+32 & 12 \end{pmatrix} +6$$

$y(t) = a e^{2t} + b e^{3t} + c e^{5t}$

$z(t) = \frac{1}{12}(-y'' + 15y' - 32y) = \frac{1}{12}(+6a)e^{2t} + 9b e^{3t} + 12c e^{5t}) =$

$= -\frac{1}{2} a e^{2t} + \frac{1}{3} b e^{3t} + c e^{5t}$

$x(t) = y' - 3y - z = (2a e^{2t} + 3b e^{3t} + 5c e^{5t}) - 3(a e^{2t} + b e^{3t} + c e^{5t})$

$- (-\frac{1}{2} a e^{2t} + \frac{1}{3} b e^{3t} + c e^{5t}) =$

$= -\frac{1}{2} a e^{2t} - \frac{1}{3} b e^{3t} + 4$