$[x_0, y_0, z_0] = [1,1,1]$

$x^2 + y^3 + z^4 = 3$ $\quad\Rightarrow\quad$ $F_1(x,y,z) = x^2 + y^3 + z^4 - 3$

$\sin(\pi x) + y^4 + z = 2$ $\qquad$ $F_2(x,y,z) = \sin(\pi x) + y^4 + z - 2$

$\Rightarrow$ • $F_1(1,1,1) = 0$ $\quad +1$ $\quad$ • $F_1, F_2 \in C^\infty(\mathbb{R}^3)$ $\quad +1$

$F_2(1,1,1) = 0$

• $\frac{\partial F_1}{\partial x}(1,1,1) = (2x)_{(1,1,1)} = 2$ $\qquad$ $\frac{\partial F_1}{\partial y}(1,1,1) = (3y^2)_{(1,1,1)} = 3$

$+2$

• $\frac{\partial F_2}{\partial x}(1,1,1) = (\pi \cos \pi x)_{(1,1,1)} = -\pi$ $\qquad$ $\frac{\partial F_2}{\partial y}(1,1,1) = (4y^3)_{(1,1,1)} = 4$

$\Rightarrow \det \begin{pmatrix} 2 & 3 \\ -\pi & 4 \end{pmatrix} = 8 + 3\pi \neq 0$ $\quad +1$

VOIF $\Rightarrow$ $\exists U \ni 1 \;\; \exists V \ni [1,1] \;\; \forall z \in U \;\; \exists! \, [x,y] \in V :$ $F_1(x,y,z) = 0$, $F_2(x,y,z) = 0$ $\quad +2$

Navíc jsou funkce $z \mapsto x(z)$, $z \mapsto y(z)$ třídy C^∞. $+1$

Máme $\quad x^2 + y^3 + z^4 - 3 = 0$ $\quad\Rightarrow\quad$ $2x x' + 3y^2 y' + 4z^3 = 0$ $\quad +3$

$\sin \pi x + y^4 + z - 2 = 0$ $\qquad$ $\pi(\cos \pi x) \cdot x' + 4y^3 y' + 1 = 0$

$(1,1,1) \Rightarrow$ $\quad 2 \cdot 1 \cdot x' + 3y' + 4 = 0 \;\Rightarrow\; y' = \frac{-4 - 2x'}{3}$

$-\pi x' + 4y' + 1 = 0 \;\Rightarrow\; -\pi x' + \frac{4}{3}(-4 - 2x') = -1$ $\quad +4$

$\pi x' + \frac{4}{3}(4 + 2x') = 1$

$3\pi x' + 16 + 8x' = 3$

$x'(3\pi + 8) = -13$

$x'(1) = \frac{-13}{3\pi + 8}$

$f = x(y+z)$ $\quad M = \{x^2+y^2+z^2 = 1,\ x^2+2y^2 \leq 1\}$

$\Rightarrow f \in C^\infty(\mathbb{R}^3)$, M kompakt, $Df = (y+z, x, x)$ +2

· $M_1 = \{x^2+y^2+z^2 = 1,\ x^2+2y^2 < 1\}$

$g = x^2+y^2+z^2-1$, $Dg = 2(x,y,z) \neq 0$ na M_1

$$\Rightarrow \begin{array}{l} y+z+\lambda x = 0 \\ x + \lambda y = 0 \\ x + \lambda z = 0 \\ x^2+y^2+z^2 = 1 \end{array} \quad \to \lambda(y-z) = 0$$

$\lambda = 0 \to y+z=0,\ x=0 \Rightarrow$ fční hodnota je $f(x,y,z) = 0$

$\lambda \neq 0 \to y = z$

$$\Rightarrow \begin{array}{l} 2y + \lambda x = 0 \\ x + \lambda y = 0 \\ \underline{x^2+2y^2 = 1} \end{array}$$

$2y + \lambda x = 0 \to 2y - \lambda^2 y = 0 \to y = 0, x = 0$ ↯

$\to \lambda^2 = 2$

$x + \lambda y = 0 \to x = -\lambda y$

$x^2 = \lambda^2 y^2 = 2y^2$

$\to 4y^2 = 1$

$y^2 = 1/4$

$y = \pm\frac{1}{2},\ z = \pm\frac{1}{2}$

+5

$\cancel{x = \sqrt{2}/4}$

$\Rightarrow \left[\mp\frac{\sqrt{2}}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right]$ $\quad \frac{2}{4} + 2\cdot\frac{1}{4} = 1$

$\left[\pm\frac{\sqrt{2}}{2}, \mp\frac{1}{2}, \mp\frac{1}{2}\right]$ $\Rightarrow$ body nejsou v M_1

· $M_2 = \{\underbrace{x^2+y^2+z^2=1}_{g_1},\ \underbrace{x^2+2y^2=1}_{g_2}\}$

$Dg_1 = 2(x,y,z)$

$Dg_2 = 2(x, 2y, 0)$

$$\begin{pmatrix} x & y & z \\ x & 2y & 0 \end{pmatrix} \sim \begin{pmatrix} x & y & z \\ 0 & y & -z \end{pmatrix}$$

· $x \neq 0 \Rightarrow$ hodnost je menší než 2 pro $y = z = 0 \Rightarrow x = \pm 1 \Rightarrow [\pm 1, 0, 0]$

· $x = 0 \Rightarrow \begin{pmatrix} 0 & y & z \\ 0 & y & -z \end{pmatrix} \sim \begin{pmatrix} 0 & y & z \\ 0 & 0 & -2z \end{pmatrix}$

$\Rightarrow y \neq 0$, pak hodnost menší než 2 pokud $z = 0$, ale takový bod neleží na M_2

+2

$\Rightarrow y = 0$ není v M_2

$y + z + \lambda x + \mu x = 0$

$x + \lambda y + \mu 2y = 0$

$x + \lambda z = 0$

$x^2 + y^2 + z^2 = 1$

$x^2 + 2y^2 = 1$

$z^2 - y^2 = 0$

$(z-y)(z+y) = 0$

$z + y = 0$

pak $f(x,y,z) = 0$

$z = y$:

$2y + x(\lambda + \mu) = 0$

$x + y(\lambda + 2\mu) = 0 \to 2\mu y = 0 \longrightarrow \dots$

$x + \lambda y = 0 \Rightarrow x = -\lambda y$

$x^2 + 2y^2 = 1$

$\dots \longrightarrow \mu = 0 \Rightarrow 2y + \lambda x = 0$

$2y - \lambda^2 y = 0$

+4

$y = 0$

$x = \pm 1$

$z = 0$

$\lambda^2 = 2$

$x^2 = \lambda^2 y^2 = 2y^2$

$\to 4y^2 = 1$

$y = \pm \frac{1}{2} \Rightarrow [\pm \frac{\sqrt{2}}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}]$

$\to \mu \neq 0 \Rightarrow y = 0, x = 0, z = 0$ ↯

Závěr: Máme body:

$[\pm \frac{\sqrt{2}}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}]$

$[\pm 1, 0, 0]$

$\Rightarrow$ maximum v: $[\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2}]$

$[-\frac{\sqrt{2}}{2}, -\frac{1}{2}, -\frac{1}{2}]$

a rovná se $\frac{\sqrt{2}}{2}$

+2

minimum v: $[-\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2}]$

$[\frac{\sqrt{2}}{2}, -\frac{1}{2}, -\frac{1}{2}]$

a rovná se $-\frac{\sqrt{2}}{2}$

Rech $\underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}}_{B}$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \overset{+3}{\sim} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right) \overset{+3}{\sim} \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array}\right)$$

$$\overset{+3}{\sim} \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -1/2 \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$$

$$\Rightarrow X = A^{-1}B = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix} \overset{+6}{=} \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$\sum (-1)^n \operatorname{tg}\big(\underbrace{\sqrt{n^p+1}-\sqrt{n^p-1}}_{a_n}\big), \quad p \geq 0$$

- $p=0$: $\sum (-1)^n \operatorname{tg}(\sqrt{2}-0)$ diverguje z nutné podmínky +1

- $p>0$: $\sqrt{n^p+1}-\sqrt{n^p-1} = \dfrac{2}{\sqrt{n^p+1}+\sqrt{n^p-1}} \xrightarrow[n\to\infty]{} 0$ +3

$$\Rightarrow \frac{\operatorname{tg}\left(\sqrt{n^p+1}-\sqrt{n^p-1}\right)}{\frac{1}{n^{p/2}}} = \frac{\operatorname{tg}\frac{2}{\sqrt{n^p+1}+\sqrt{n^p-1}}}{\frac{2}{\sqrt{n^p+1}+\sqrt{n^p-1}}} \cdot 2 \cdot \frac{n^{p/2}}{\sqrt{n^p+1}+\sqrt{n^p-1}} \longrightarrow 1 \cdot 2 \cdot \frac{1}{\sqrt{1}+\sqrt{1}} = 1$$

~~+4~~ +3

+3

$\Rightarrow \sum \operatorname{tg} a_n$ konv. $\Leftrightarrow \sum \frac{1}{n^{p/2}}$ konv. $\Leftrightarrow p/2 > 1$
$p > 2$ ~~+3~~ +1

~~+3~~ +1

- $p \in (0, 2\rangle$: $a_n \searrow 0 \Rightarrow \operatorname{tg} a_n \searrow 0 \Rightarrow \sum (-1)^n \operatorname{tg} a_n$ konverguje dle Leibnize

~~+4~~

+3