$$\underbrace{\cos(xz - 1 + ye^{z}) - ze^{xy}}_{F} = 0$$

$[x_0, y_0, z_0] = [1, 0, 1]$

- $F(1,0,1) = \cos 0 - 1 \cdot 1 = 0$ (+1)
- $F \in C^{\infty}(\mathbb{R}^3)$ (+1)
- $\left(\dfrac{\partial F}{\partial y}\right)\Big|_{[1,0,1]} = \left[-\sin(xz - 1 + ye^{z})(e^{z}) - ze^{xy}x\right]_{[1,0,1]} =$

$= -1 \cdot e^{0} \cdot 1 = -1 \neq 0$ (+2)

$\Rightarrow$ VOIF funguje, udáva $\varphi : U \ni [1,1] \to V \ni 0$, ž. $\varphi \in C^{\infty}(U)$ (+2)

- $\varphi(1,1) = 0$
- $F(x, \varphi(x,z), z) = 0, \ (x,z) \in U$

$\dfrac{d}{dx}$: $-\sin(xz - 1 + ye^{z})(z + y_x e^{z}) - ze^{xy}(y + xy_x) = 0$

$\Rightarrow -\sin 0 \ (\ldots\ldots) - 1(0 + 1 \cdot y_x) = 0$ (3.5)

$\Rightarrow y_x(1,1) = 0$

$\dfrac{d}{dz}$: $-\sin(xz - 1 + ye^{z})(\ldots\ldots) - e^{xy} - ze^{xy}(xy_z) = 0$

$\Rightarrow -\sin 0 \ (\ldots) - 1 - 1 \cdot 1 \cdot 1 y_z = 0$ (+3.5)

$\Rightarrow y_z(1,1) = -1$

Teda $T(x,z) = \varphi(1,1) + \varphi_x(1,1)(x-1) + \varphi_z(1,1)(z-1)$ (+2)

$= 0 + 0 + (-1)(z-1) = 1 - z$

$f(x,y,z) = x + 2y + 3z \quad M = \{ x^2+y^2+z^2 \leq 1,\ x+y \geq 1 \} \quad f \in C(\mathbb{R}^3),\ M$ kompakt (+2)

- $\nabla f = (1, 2, 3) \neq 0$ na $\mathbb{R}^3 \Rightarrow$ na $M_1 = \{x^2+y^2+z^2<1,\ x+y>1\}$ není podezřelý bod (+1)

- $M_2 = \{x^2+y^2+z^2=1,\ x+y=1\} \Rightarrow N = \{x^2+(1-x)^2+z^2=1\}$

$$h(x,z) = x + 2(1-x) + 3z$$
$$= 2 - x + 3z$$

$\nabla g = 2(2x-1, z) = 0 \iff [x, z] = [1/2, 0] \notin N$

$\nabla h = (-1, 3)$

$-1 + \lambda(2x-1) = 0 \quad / \cdot z$

$3 + \lambda z = 0 \quad / (2x-1)$

$-z - 3(2x-1) = 0$

$z = -3(2x-1)$

$x^2 + (1-x)^2 + z^2 = 1$

$2x^2 - 2x + 9(2x-1)^2 = 0$ (+4)

$2x^2 - 2x + 9(4x^2 - 4x + 1) = 0$

$38x^2 - 38x + 9 = 0$

$$x_{1,2} = \frac{38 \pm \sqrt{38^2 - 4 \cdot 38 \cdot 9}}{2 \cdot 38} = \frac{38 \pm \sqrt{38(38-36)}}{2 \cdot 38} =$$

$$= \frac{38 \mp \sqrt{38}\sqrt{2}}{2 \cdot 38} = \frac{\sqrt{38} \pm \sqrt{2}}{2\sqrt{38}}$$

$y_{1,2} = 1 - x_{1,2}, \quad z_{1,2} = -3(2x_{1,2} - 1)$

- $M_3 = \{x^2+y^2+z^2<1,\ x+y=1\}$

$1 + \lambda \cdot 1 = 0$

$2 + \lambda \cdot 1 = 0$

$3 + \lambda \cdot 0 = 0$ NŘ

$x + y = 1$

(+2)

$M_4 = \{x^2+y^2+z^2=1,\ x+y>1\}$

$1 + \lambda x = 0 \quad x = -\frac{1}{\lambda}$

$2 + \lambda y = 0 \quad y = -\frac{2}{\lambda}$

$3 + \lambda z = 0 \quad z = -\frac{3}{\lambda}$

$x^2 + y^2 + z^2 = 1 \Rightarrow \left(\frac{1}{\lambda}\right)^2 + \frac{4}{\lambda^2} + \frac{9}{\lambda^2} = 1$

$\lambda^2 = 14$

$\lambda = \pm\sqrt{14}$

$[x, y, z] = \left[\frac{2}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right] \notin M_4$ (+3)

~~Závěr:~~

<u>Zaveď</u>: $t := \dfrac{\sqrt{38}+\sqrt{2}}{2\sqrt{38}}$, $s = \dfrac{\sqrt{38}-\sqrt{2}}{2\sqrt{38}}$

· $\alpha = [\, t,\ 1-t,\ -3(2t-1)\,]$

$f(\alpha) = t + 2(1-t) - 9(2t-1) = 11 - 19t$

+3

· $\beta = [\, s,\ 1-s,\ -3(2s-1)\,]$

$f(\beta) = 11 - 19s$

$t > s \Rightarrow f(\alpha) < f(\beta)$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & a & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -a \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 6 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 4 & 1 & -6 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 8-6 & -4 & 1 \end{array}\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 8-6 & -4 & 1 \end{pmatrix}$$

+4 +4

$$\Rightarrow B^{-1}A^{-1} = \cancel{\begin{pmatrix} 8a-a6 & 4a & -a \\ -2 & 1 & 0 \\ 8-6 & -4 & 1 \end{pmatrix}} \quad a \quad \det B^{-1}A^{-1} = \frac{1}{\det A} \cdot \frac{1}{\det B} = 1$$

+3

$$= \begin{pmatrix} 1 & 0 & -a \\ -2 & 1 & 2a \\ 8-6 & -4 & -a(8-6)+1 \end{pmatrix}$$

+4

$\sum (x+1)^n \frac{\sqrt{n}}{n+1}$, $x \in \mathbb{R}$

(• $x=-1$ ... A.K.)

• podílové kritérium: $\frac{|x+1|^{n+1}\sqrt{n+1}}{n+2} \cdot \frac{n+1}{\sqrt{n}} \cdot \frac{1}{|x+1|^n} = |x+1| \frac{n+1}{n+2}\sqrt{\frac{n+1}{n}} \to |x+1|$

$\Rightarrow$ • $x \in (-2,0)$ A.K.

• $|x+1|>1$ divergence

• $x=0$: $\frac{\sqrt{n}}{n+1}$ srovnám s $\frac{1}{\sqrt{n}}$, jelikož $\sum \frac{1}{\sqrt{n}}$ diverguje, diverguje i řada $\sum \frac{\sqrt{n}}{n+1}$

• $x=-2$: $\sum (-1)^n \frac{\sqrt{n}}{n+1}$: $\frac{\sqrt{n}}{n+1} \geq \frac{\sqrt{n+1}}{n+2}$

$+ \to 0$

$\left(| \ | \leq \frac{1}{\sqrt{n}} \to 0 \right)$

$$\sqrt{n}(n+2) \geq (n+1)\sqrt{n+1}$$
$$n(n+2)^2 \geq (n+1)^2(n+1)$$
$$n(n^2+4n+4) \geq n^3+3n^2+3n+1$$
$$n^3+4n^2+4n \geq n^3+3n^2+3n+1$$
$$n^2+n \geq 1 \ \checkmark$$

$\Rightarrow$ $\frac{\sqrt{n}}{n+1} \to 0$ z Leibnize řada konverguje

A.K. pro $(-2,0)$: 5 | $|x+1|>1$ nek. - argument?

bod 0 : 4

bod -2 : 6 2 $\to 0$ 3 1