

$$1. \quad n(3^{1/n} - 4^{1/n}) = \frac{3^{1/n} - 1}{1/n} + \frac{1 - 4^{1/n}}{1/n} \rightarrow \log 3 + 1 - \log 4 = \log 3 - \log 4$$

$$\frac{3^x - 1}{x} \xrightarrow{x \rightarrow 0} \log 3$$

$$\frac{1 - 4^x}{x} \xrightarrow{x \rightarrow 0} -\log 4$$

přičítání jechatky ... 5

$$\frac{3^x - 1}{x} \dots 4$$

$$\frac{1 - 4^x}{x} \dots 4$$

$$= \log 3$$

závěr ... 2

$$2. \quad (\cos 2\pi x)^{\frac{1}{\sin 2\pi x}} = e^{\frac{1}{\sin 2\pi x} \log(\cos 2\pi x)} \xrightarrow{x \rightarrow 1} e^{-2}$$

$$f(x) = \frac{(x-1)^2}{\sin 2\pi x} \cdot \frac{\log \cos 2\pi x}{\cos 2\pi x - 1} \cdot \frac{\cos 2\pi x - 1}{(x-1)^2} \xrightarrow{x \rightarrow 1} \frac{1}{\pi^2} \cdot 1 \cdot \frac{-4\pi^2}{2} = -2$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\sin 2\pi x} \stackrel{0/0}{=} \lim_{x \rightarrow 1} \frac{1}{\pi \cos 2\pi x} = \frac{-1}{\pi}$$

$$\lim_{x \rightarrow 1} \frac{\cos 2\pi x - 1}{(x-1)^2} \stackrel{0/0}{=} \lim_{x \rightarrow 1} \frac{(-\sin 2\pi x) \cdot 2\pi}{2(x-1)} \stackrel{0/0}{=} \lim_{x \rightarrow 1} \frac{(-\cos 2\pi x) \cdot 4\pi^2}{2} = \frac{-4\pi^2}{2}$$

(*) : $\cos 2\pi x \neq 1$ na $P(1, \frac{2}{3})$

• přeepis ... 3

• sin ... 4

• cos ... 4

• výpočet $f(x)$... 3

• závěr ... 1

$$3. f(x) = \begin{cases} x^{(x^c)} - 1, & x \in (0, 1) \\ \sin c(x-1), & x \in (1, \infty) \end{cases}$$

$A \in c \in \mathbb{R}$

$$\lim_{x \rightarrow 1^+} f(x) = \sin(c \cdot 0) = \sin 0 = 0$$

$$f(1) = \lim_{x \rightarrow 1} (e^{x^c \log x} - 1) = \lim_{x \rightarrow 1} (e^{e^{c \log x} \log x} - 1) = e^{e^{c \cdot 0} \cdot 0} - 1 = 0$$

Tedy f spojita na $(0, \infty)$ pro libovolné $c \in \mathbb{R}$.

$$f'(x) = \begin{cases} (e^{e^{c \log x} \log x} - 1)' = e^{e^{c \log x} \log x} (e^{c \log x} \frac{c}{x} \log x + e^{c \log x} \cdot \frac{1}{x}), & x \in (0, 1) \\ (\cos c(x-1)) \cdot c, & x \in (1, \infty) \end{cases}$$

$$f'_+(1) = \lim_{x \rightarrow 1^+} f'(x) = c \cdot 1 = c$$

$$f'_-(1) = \lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} (x^{(x^c)} (x^c) (c \frac{\log x}{x} + \frac{1}{x})) = 1 \cdot 1 \cdot (c \cdot \frac{0}{1} + 1) = 1$$

\Rightarrow pro $c = 1$ je $f'(1) = 1$, jinak neexistuje

• spojita f_{-} ... 3

• f' upravo ... 3

• f' ulavo ... 6

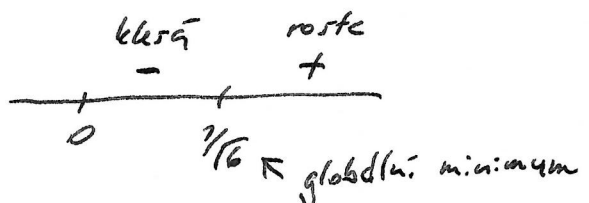
• Závěr ... 3

4. $f(x) = \log\left(2x + \frac{1}{3x}\right)$, $D_f = (0, \infty)$, f spojitelno na $(0, \infty)$

$\lim_{x \rightarrow 0^+} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$

$f'(x) = \frac{1}{2x + \frac{1}{3x}} \cdot \left(2 - \frac{1}{3x^2}\right) = \frac{3x}{6x^2 + 1} \cdot \left(\frac{6x^2 - 1}{3x^2}\right) = \frac{6x^2 - 1}{x(6x^2 + 1)}$

$f' = 0 \Leftrightarrow 6x^2 - 1 = 0 \Leftrightarrow x^2 = \frac{1}{6} \Leftrightarrow x = \frac{1}{\sqrt{6}}$



$f''(x) = \left(\frac{6x^2 - 1}{6x^3 + x}\right)' = \frac{1}{(6x^3 + x)^2} [12x(6x^2 + 1) - (6x^2 - 1)(18x^2 + 1)] =$

$= \frac{1}{(6x^3 + x)^2} (72x^3 + 12x^2 - 108x^4 - 6x^2 + 18x^2 + 1)$

$= -11 (-36x^3 + 29x^2 + 1)$

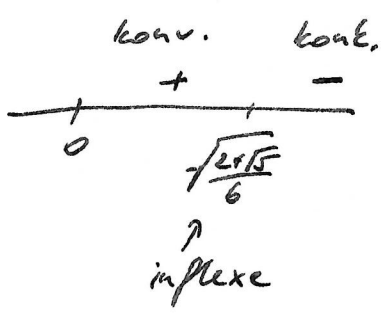
$= \frac{-1}{(6x^3 + x)^2} (36x^3 - 29x^2 - 1)$

$36y^2 - 29y - 1 = 0 \Rightarrow y_{1,2} = \frac{29 \pm \sqrt{29^2 + 4 \cdot 36}}{72} = \frac{29 \pm 72\sqrt{5}}{72}$

$\sqrt{29^2 + 4 \cdot 36} = (4 \cdot 6)^2 + 4 \cdot 6^2 = 4^2 \cdot 6^2 + 4 \cdot 6^2 = 4 \cdot 6^2 (1 + 4) = 5 \cdot 4 \cdot 6^2$

$= \frac{2 \pm \sqrt{5}}{6}$, zrajmo nas korjen $\frac{2 + \sqrt{5}}{6}$, tj. $x^2 = \frac{2 + \sqrt{5}}{6} \Rightarrow$

$x = \sqrt{\frac{2 + \sqrt{5}}{6}}$

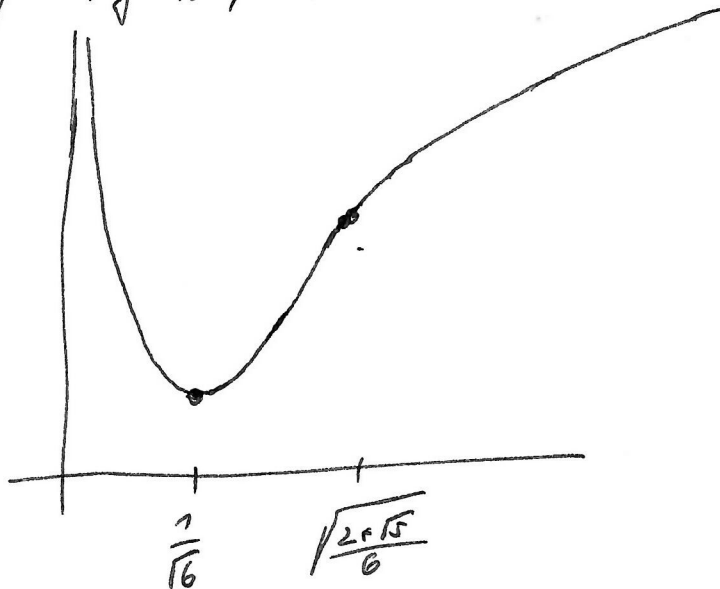


asimptote: $\lim_{x \rightarrow \infty} \frac{f(x)}{x} = 0$ & $\lim_{x \rightarrow \infty} (f(x) - 0 \cdot x) = \infty$

$\left[0 \leq \frac{f(x)}{x} \leq \frac{\log(2x+1)}{x} \rightarrow 0 \right]_{x \rightarrow \infty}$

\Rightarrow asimptota ne obstoje

$$H_f = \left(f\left(\frac{1}{\sqrt{6}}\right), \infty \right)$$



$$f\left(\frac{1}{\sqrt{6}}\right) > 0? :$$

$$2 \cdot \frac{1}{\sqrt{6}} + \frac{1}{3\sqrt{6}} = \frac{2}{\sqrt{6}} + \frac{\sqrt{6}}{3} = \frac{6+6}{3\sqrt{6}} = \frac{12}{3\sqrt{6}} > 1$$

$$12 > 3\sqrt{6}$$

$$4 > \sqrt{6}$$

$$16 > 6 \checkmark$$

• D_f ... ~~SNS~~ 1

• \liminf ... SNS 1

• spojitost ... 1

• f' ... 2

• monotonie 2

• extrémy , ry 1

• f'' ... 2

• konvexita ... 2

• asymptota ... 1

• graf ... 2