1. $(\log n)^{(\sqrt{n})^{1/n}} = e^{(\sqrt{n})^{1/n} \log \log n} = e^{e^{\frac{1}{n} \log \sqrt{n}} \log \log n} \to \infty$

- $\frac{1}{n} \log \sqrt{n} = \frac{1}{2n} \log n \longrightarrow 0$
- $e^{\frac{1}{n} \log \sqrt{n}} \to 1$
- $\log(\log n) \to \infty$

1. přepis ... 4
2. přepis ... 4
$\frac{\log n}{n}$ ... 4
$\log(\log n)$ ... 2
zdův ... 1

2. $\left(\sqrt{x+2\sqrt{x}} - \sqrt{x}\right)^{\sqrt{x}} = e^{\underbrace{\sqrt{x} \log \overbrace{\left(\sqrt{x+2\sqrt{x}} - \sqrt{x}\right)}^{g(x)}}_{f(x)}}$

- $\sqrt{x+2\sqrt{x}} - \sqrt{x} = \frac{2\sqrt{x}}{\sqrt{x+2\sqrt{x}} + \sqrt{x}} = \frac{2}{\sqrt{1+2\frac{1}{\sqrt{x}}} + 1} \to 1$

- $f(x) = \sqrt{x}\, \frac{\log g(x)}{g(x)-1} (g(x)-1) = \frac{\log g(x)}{g(x)-1} \sqrt{x} \cdot \frac{\sqrt{x} - \sqrt{x+2\sqrt{x}}}{\sqrt{x+2\sqrt{x}} + \sqrt{x}} =$

$= \frac{\log g(x)}{g(x)-1} \frac{-2\sqrt{x}}{\left(\sqrt{x+2\sqrt{x}} + \sqrt{x}\right)^2} \sqrt{x} = \frac{\log g(x)}{g(x)-1} \frac{(-2\cdot\cancel{x})}{\left(\sqrt{1+\frac{2}{\sqrt{x}}} + 1\right)^2} \xrightarrow[x\to\infty]{} 1 \cdot \frac{(-2)}{4}$

$\frac{2\sqrt{x}}{\sqrt{x+2\sqrt{x}} + \sqrt{x}} < 1$ $\Rightarrow$ splněna podmínka (P)

$\sqrt{x} < \sqrt{x+2\sqrt{x}}$, $x > 0$

zdův: $e^{f(x)} \xrightarrow[x\to\infty]{} e^{-1/2}$

přepis ... 2
úprava g ... 3
log ... 3
(P) ... 3
úprava zlomku ... 3

zdův ... 1

3. $f(x) = (x-1)\arcsin \underbrace{\frac{2x}{x^2+1}}_{g(x)}$

$g'(x) = 2\,\frac{x^2+1-2x^2}{(x^2+1)^2} = 2\,\frac{1-x^2}{(1+x^2)^2} \Rightarrow g$ má extrémy v ± 1

$H_g = \langle g(-1), g(1)\rangle = \langle -1, 1\rangle$, neboť $\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to\infty} g(x) = 0$

Tedy · $g(x) \in (-1,1)$, $x \in \mathbb{R}\setminus\{-1,1\}$

· $g(-1) = -1$, $g(1) = 1$

· $f'(x) = \arcsin\frac{2x}{x^2+1} + (x-1)\frac{1}{\sqrt{1-\left(\frac{2x}{x^2+1}\right)^2}}\cdot 2\cdot\frac{1-x^2}{(1+x^2)^2} =$

$= \arcsin g(x) + (x-1)\frac{2}{\sqrt{\frac{x^4+2x^2+1-4x^2}{(x^2+1)^2}}}\cdot\frac{1-x^2}{(1+x^2)^2} =$

$= \arcsin g(x) + 2(x-1)\frac{1-x^2}{\sqrt{(x^2-1)^2}}\cdot\frac{1}{1+x^2}$

$= \arcsin g(x) + \frac{2(x-1)(1-x^2)}{(1+x^2)\,|x^2-1|}$, $x \neq \pm 1$

· f spojitá na $\mathbb{R}$ $\Rightarrow$ $f'(1) = \lim\limits_{x\to 1} f'(x) = \arcsin g(1) + 0 =$

$\frac{1-x^2}{|x^2-1|}$ je omezená

$= \arcsin 1 = \pi/2$

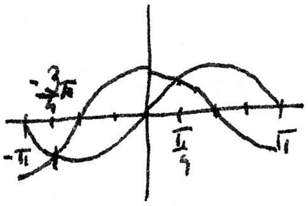
· $f'_-(-1) = \lim\limits_{x\to -1-} f'(x) = \arcsin(-1) + \frac{2(-2)}{2}\cdot(-1) = -\frac{\pi}{2} + 2$

· $f'_+(-1) = \lim\limits_{x\to -1+} f'(x) = \arcsin(-1) + \frac{2\cdot(-2)}{2}\cdot 1 = -\frac{\pi}{2} - 2$

D_f ... 4 spojitost f ... 1 $f'(1)$ ... 3

f' mimo ± 1 ... 3 $f'_-(-1)$ ... 2 $f'_+(-1)$ ... 2

5. $f(x) = \sin x + |\cos x - \sin x|$, $x \in \langle -\frac{3}{4}\pi, \pi \rangle$.



$$f(x) = \begin{cases} \cos x & \dots x \in \langle -\frac{3\pi}{4}, \frac{\pi}{4} \rangle \\ 2\sin x - \cos x & \dots x \in \langle \frac{\pi}{4}, \pi \rangle \end{cases}$$

$$f'(x) = \begin{cases} -\sin x & \dots x \in (-\frac{3\pi}{4}, \frac{\pi}{4}) \\ 2\cos x + \sin x & \dots x \in (\frac{\pi}{4}, \pi) \end{cases}$$

$f'_+(-\frac{3\pi}{4}) = +\frac{1}{\sqrt{2}}$, $f'_-(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$

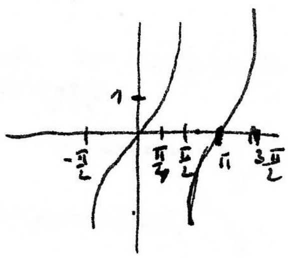
$f'_+(\frac{\pi}{4}) = \frac{3}{\sqrt{2}}$, $f'_-(\pi) = -2$

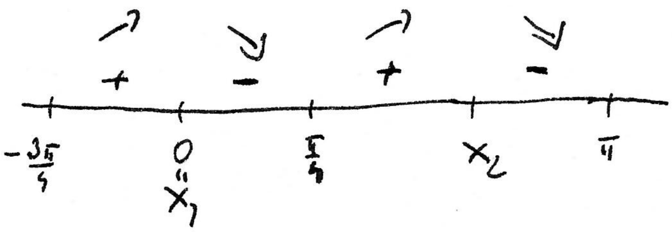
$f'(x) = 0 \Rightarrow x_1 = 0$ nebo $2\cos x + \sin x = 0$, $x \in (\pi/4, \pi)$

$2 + \operatorname{tg} x = 0$

$\operatorname{tg} x = -2$

$x_2 = \operatorname{arctg}(-2) + \pi$





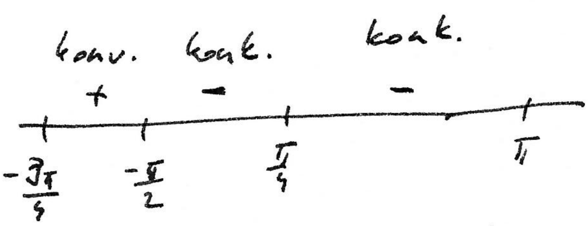
$$f''(x) = \begin{cases} -\cos x & \dots x \in (-\frac{3\pi}{4}, \pi/4) \\ -2\sin x + \cos x & \dots x \in (\pi/4, \pi) \end{cases}$$

$f''(x) = 0 \Rightarrow x_3 = -\frac{\pi}{2}$ nebo $-2\sin x + \cos x = 0$

$-2\operatorname{tg} x + 1 = 0$

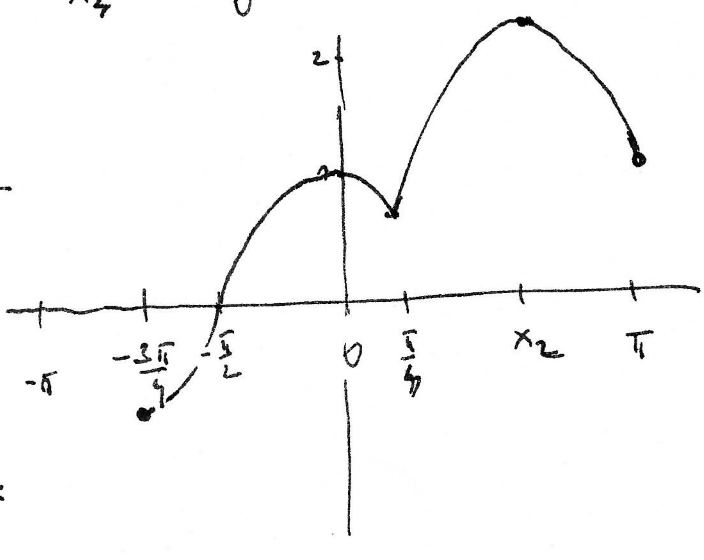
$\operatorname{tg} x = 1/2$

$x_4 = \operatorname{arctg}\frac{1}{2} < \frac{\pi}{4}$



hodnoty:

x	$f(x)$
$-\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$
$-\frac{\pi}{2}$	0
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\pi - \operatorname{arctg} 2$	$2 \cdot \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \sqrt{5}$
π	1



$f(x_2)$: $\operatorname{tg} x_2 = -2$

$$\frac{\sin x_2}{\cos x_2} = -2$$

$\sin x_2 = -2 \cos x_2$

$\sin^2 x_2 = 4(1 - \sin^2 x^2) = 4 - 4\sin^2 x^2$

$5 \sin^2 x_2 = 4$

$\sin x_2 = \frac{2}{\sqrt{5}}$

$\cos x_2 = -\sqrt{1 - \sin^2 x_2} = -\sqrt{1 - \frac{4}{5}} =$

$= -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}}$

rozdělení na intervaly ... 2
spojitost

f' normální ... 2

f' jednostranné ... 2

monotonie, extrémy ... 2,5

f'' 2

konvexita ... 2,5

graf 2