1. $\left(1-\cos\frac{2n}{n^2+1}\right)\log\left(3^{n^2}+1\right) = \underbrace{\frac{1-\cos\frac{2n}{n^2+1}}{\left(\frac{2n}{n^2+1}\right)^2}}_{\ell(n)} \left(\frac{2n}{n^2+1}\right)^2 \cdot \log 3^{n^2}\left(1+\frac{1}{3^{n^2}}\right) =$

$= 4\cdot\ell(n)\cdot\frac{n^2}{(n^2+1)^2}\cdot n^2\log 3 + 4\,\ell(n)\cdot\frac{n^2}{(n^2+1)^2}\cdot\log\left(1+\frac{1}{3^{n^2}}\right)$

$= \ell(n)\cdot 4\cdot\frac{n^4}{n^4+2n^2+1}\log 3 + 4\,\ell(n)\,\frac{n^2}{n^4+2n^2+1}\log\left(1+\frac{1}{3^{n^2}}\right)$

$\longrightarrow \frac{1}{2}\cdot 4\cdot 1\cdot\log 3 + 0 = 2\cdot\log 3$

cos ... 5

log ... 5

úprava ... 3

závěr ... 2

2. $\left(\sin\sqrt{x+1} - \sin\sqrt{x}\right)\left(\sqrt[3]{x+\sqrt[3]{x^2}} - \sqrt[3]{x}\right) =$

$$= 2\cos\frac{\sqrt{x+1}+\sqrt{x}}{2}\sin\frac{\sqrt{x+1}-\sqrt{x}}{2}\cdot\frac{x+\sqrt[3]{x^2}-x}{(x+\sqrt{x})^{2/3}+(x+\sqrt{x})^{1/3}x^{1/3}+x^{2/3}} =$$

$$= 2\cos\frac{\sqrt{x+1}+\sqrt{x}}{2}\sin\frac{1}{2}\frac{1}{\sqrt{x+1}+\sqrt{x}}\cdot \text{———} \;// \;\text{———}$$

$$= 2\cos\frac{\sqrt{x+1}+\sqrt{x}}{2}\,\frac{\sin\frac{1}{2}\frac{1}{\sqrt{x+1}+\sqrt{x}}}{\frac{1}{2}\frac{1}{\sqrt{x+1}+\sqrt{x}}}\,\frac{1}{2(\sqrt{x+1}+\sqrt{x})}\cdot\frac{\sqrt[3]{x^2}}{(x+\sqrt{x})^{2/3}+(x+\sqrt{x})^{1/3}x^{1/3}+x^{2/3}}$$

$$= \underbrace{2\cos\frac{\sqrt{x+1}+\sqrt{x}}{2}}_{\text{omezen\'a}}\;\underbrace{\frac{\sin -//-}{-//-}}_{\to 1}\;\underbrace{\frac{1}{2}\frac{1}{\sqrt{x+1}+\sqrt{x}}}_{\to 0}\cdot\underbrace{\frac{x^{2/3}}{(x+\sqrt{x})^{2/3}+(x+\sqrt{x})^{1/3}x^{1/3}+x^{2/3}}}_{\to 1/3}$$

$$\xrightarrow[x\to\infty]{} 0$$

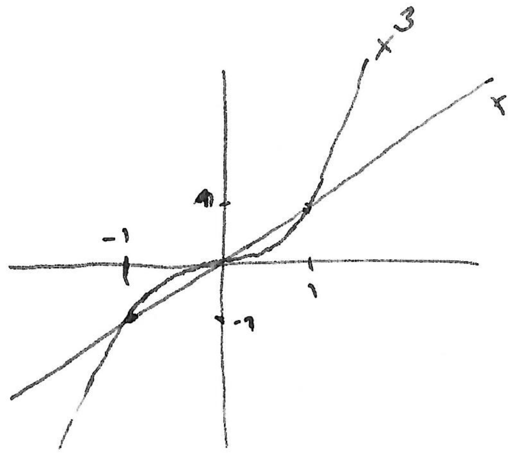
sin - sin 5

$\sqrt[3]{\ } - \sqrt[3]{\ }$ ... 5

$\frac{\sin x}{x}$ 5

zdůvod 3

3. $f(x) = \max\{x, x^3\}$, $x \in \mathbb{R}$



- $f(x) = \begin{cases} x & \dots\ x \in (-\infty, -1\rangle \\ x^3 & \dots\ x \in (-1, 0\rangle \\ x & \dots\ x \in (0, 1\rangle \\ x^3 & \dots\ x \in (1, \infty) \end{cases}$ $\quad f'(x) = \begin{cases} 1 & \dots\ x \in (-\infty, -1) \\ 3x^2 & \dots\ x \in (-1, 0) \\ 1 & \dots\ x \in (0, 1) \\ 3x^2 & \dots\ x \in (1, \infty) \end{cases}$

- f spojitá na $\mathbb{R}$

- $f'_-(-1) = 1$, $f'_+(-1) = 3$
- $f'_-(0) = 0$, $f'_+(0) = 1$
- $f'_-(1) = 1$, $f'_+(1) = 3$

} derivace f v bodech $-1, 0, 1$ neexistuje

rozepis f 3

f' mimo $\{-1, 0, 1\}$ .. 4

f spojitá ... 2

$f'(-1)$ 2

$f'(0)$ ... 2

$f'(1)$ ... 2

5. $f(x) = \dfrac{x}{\sqrt{|x^2-1|}}$ $\Rightarrow$ $D(f) = \mathbb{R} \setminus \{-1,1\}$, f lichá, f spojitá na $\mathbb{R}\setminus\{-1,1\}$

- $\lim_{x\to -\infty} f(x) = -1$, $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to 1+} f(x) = \infty$, $\lim_{x\to 1-} f(x) = -\infty$

- $f(x) = \begin{cases} \dfrac{x}{\sqrt{x^2-1}}, & x \in (-\infty,-1)\cup(1,\infty) \\ \dfrac{x}{\sqrt{1-x^2}}, & x\in(-1,1) \end{cases}$

- $f'(x) = $

$\left(x(x^2-1)^{-1/2}\right)' = (x^2-1)^{-1/2} + x(-\frac{1}{2})(x^2-1)^{-3/2}2x =$
$= (x^2-1)^{-3/2}\left((x^2-1) - x^2\right) = (-1)(x^2-1)^{-3/2}$, $x \in (-\infty,-1)\cup(1,\infty)$

$\left(x(1-x^2)^{-1/2}\right)' = (1-x^2)^{-1/2} + x(-\frac{1}{2})(1-x^2)^{-3/2}(-2x)$
$= (1-x^2)^{-3/2}\left((1-x^2)+x^2\right) = (1-x^2)^{-3/2}$, $x\in(-1,1)$

mon. ↘ ↗ ↘

f': <0 | -1 | >0 | 1 | <0 $H_f = \mathbb{R}$

- $f''(x) = \begin{cases} 3x(x^2-1)^{-5/2} & \dots\ x\in(-\infty,-1)\cup(1,\infty) \\ 3x(1-x^2)^{-5/2} & \dots\ x\in(-1,1) \end{cases}$

konv. konk. konk. konv. konv.

f'': <0 | -1 | <0 | 0 | >0 | 1 | >0 ~ 0 je inflexe

- asymptoty: $y(x) = 1$ ~ ∞
  $y(x) = -1$ ~ $-\infty$

D_f ... 1
limity ... 1
spojitost ... 1
f' ... 2
mon. ... 2
H_f ... 1
f'' ... 2
konv ... 2
asympt. ... 1
graf ... 2