1. $\dfrac{\log(2^n + \sin n + n^3)}{\log(n^{12} + 5^n)} = \dfrac{\log 2^n\left(1 + \frac{\sin n}{2^n} + \frac{n^3}{2^n}\right)}{\log 5^n\left(1 + \frac{n^{12}}{5^n}\right)} =$

$= \dfrac{n\log 2 + \log\left(1 + \frac{\sin n}{2^n} + \frac{n^3}{2^n}\right)}{n\log 5 + \log\left(1 + \frac{n^{12}}{5^n}\right)} = \dfrac{\log 2 + \frac{1}{n}\log\left(1 + \frac{\sin n}{2^n} + \frac{n^3}{2^n}\right)}{\log 5 + \frac{1}{n}\log\left(1 + \frac{n^{12}}{5^n}\right)}$

$\xrightarrow[n\to\infty]{} \dfrac{\log 2}{\log 5}$

vytknutí předcházejícího členu ... 6

rozložení na součet log ... 4

podělení n ... 4

závěr ... 1

2.

$$\lim_{x\to 0} \frac{e^x + e^{-x} - 2}{e^x - x - \cos x} \overset{0/0}{=} \lim_{l'H\ x\to 0} \frac{e^x - e^{-x}}{e^x - 1 + \sin x} \overset{0/0}{=} \lim_{l'H\ x\to 0} \frac{e^x + e^{-x}}{e^x + \cos x} = \frac{2}{1+1} = 1$$

první derivace ... 7

druhé derivace ... 6

závěr ... 2

3. $f(x) = x^2 [x]$

- $x \notin \mathbb{Z} \Rightarrow [x]' = 0 \Rightarrow f'(x) = 2x[x] + x^2[x]' = 2x[x]$

- $x = 0 \Rightarrow f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} x[x] = 0$

- $a \in \mathbb{Z} \setminus \{0\}$: $f'_+(a) = \lim_{x \to a+} \frac{x^2[x] - a^2[a]}{x - a} = \lim_{x \to a} \frac{x^2[a] - a^2[a]}{x - a} =$

$= [a] \lim_{x \to a} (x + a) = 2a[a]$

$f'_-(a) = \lim_{x \to a-} \frac{x^2[x] - a^2[a]}{x - a} = \lim_{x \to a-} \frac{x^2([a] - 1) - a^2[a]}{x - a} =$

$= \lim_{x \to a-} \left([a] \frac{x^2 - a^2}{x - a} - \frac{x^2}{x - a} \right) \overset{A.L.}{=} +\infty$

- $x \notin \mathbb{Z}$ 5
- $x = 0$ 4
- $a \in \mathbb{Z} \setminus \{0\}$: $f'_+(a)$ 3
  $f'_-(a)$ 5

5.

$$f(x) = \begin{cases} x e^{-|1-\frac{1}{x}|} \dots x \neq 0 \\ 0 \dots x = 0 \end{cases} = \begin{cases} x e^{-(1-\frac{1}{x})} \dots x \in (-\infty, 0) \cup (1, \infty) \\ 0 \dots x = 0 \\ x e^{(1-\frac{1}{x})} \dots x \in \langle 0, 1 \rangle \end{cases}$$

$\Rightarrow \lim_{x \to 0} f(x) = 0 \Rightarrow f$ spojitá na $\mathbb{R}$

$$f'(x) = \begin{cases} \left(e^{-1} x e^{1/x}\right)' = e^{-1}\left(e^{1/x} + x e^{1/x}\left(-\frac{1}{x^2}\right)\right) = e^{-1} e^{1/x}\left(1 - \frac{1}{x}\right) \\ x \in (-\infty, 0) \cup (1, \infty) \\ \left(e x e^{-1/x}\right)' = e\left(e^{-1/x} + x e^{-1/x} \frac{1}{x^2}\right) = e e^{-1/x}\left(1 + \frac{1}{x}\right), \ x \in (0, 1) \end{cases}$$

$$f'_-(0) = \lim_{x \to 0_-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0_-} e^{-1} e^{1/x} = 0$$

$$f'_+(0) = \lim_{x \to 0_+} \text{---''---} = \lim_{x \to 0_+} e e^{-1/x} = 0 \Big\} \Rightarrow f'(0) = 0$$

$f'(1)$: $f'_-(1) = \lim_{x \to 1_-} f'(x) = e e^{-1/1}(1 + 1/1) = 2$ $\Rightarrow f'(1)$ neexistuje

$f'_+(1) = \lim_{x \to 1_+} f'(x) = e^{-1} e^{1/1}(1 - 1/1) = 0$

mon.: f': $+$ 0 $+$ $\nexists$ $+$ — f spojitá $\Rightarrow f$ rostoucí na $\mathbb{R}$

$\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = -\infty \Rightarrow H_f = \mathbb{R}$

$$f''(x) = \begin{cases} e^{-1}\left(e^{1/x}\left(-\frac{1}{x}\right)^2\left(1 - \frac{1}{x}\right) + e^{1/x}\frac{1}{x^2}\right) = e^{-1} e^{1/x} \cdot \frac{1}{x^2}\left(-1 + \frac{1}{x} + 1\right) = e^{-1} e^{1/x} \frac{1}{x^3} \\ x \in (-\infty, 0) \cup (1, \infty) \\ e\left(e^{-1/x} \frac{1}{x^2}\left(1 + \frac{1}{x}\right) + e^{-1/x}\left(-\frac{1}{x^2}\right)\right) = e e^{-1/x} \cdot \frac{1}{x^3}, \ x \in (0, 1) \end{cases}$$

konv.: f'': $-$ 0 $+$ $\nexists$ $+$ — konk. | konv. | konv.

$f'' < 0$ na $(-\infty, 0)$

$f'' > 0$ na $(0, \infty)$

$\Big\} \Rightarrow$ v 0 je inflexe

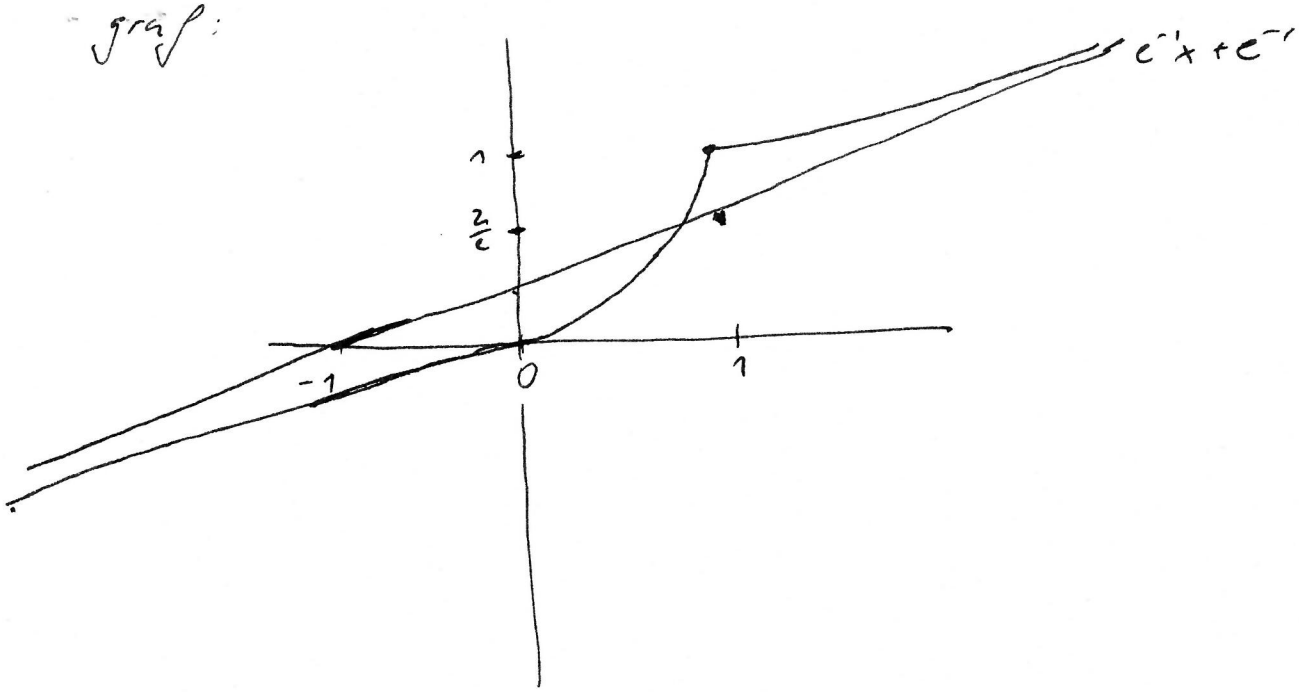
tečna do $[0, f(0)]$ je $t(x) = 0$

asymptoty: $\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} e^{-1} e^{1/x} = e^{-1}$

$$\lim_{x \to \pm\infty} f(x) - e^{-1} x = \lim_{x \to \pm\infty}\left(e^{-1} x e^{1/x} - e^{-1} x\right) = \lim_{x \to \pm\infty} e^{-1} \frac{e^{1/x} - 1}{1/x} = e^{-1}$$

asymptota v $\pm\infty$ má tvar $a(x) = e^{-1} x + e^{-1}$

graf:



vpsitovt f ... 1

f' ... 2

monotonic 2

$\#_f$ ... 1

f'' ... 2

konvexita, inflexe ... 2+1

asymptoty ... 2

graf ... 2