1. $\underbrace{\sqrt{n^2+n} - \sqrt[3]{n^3+n}}_{a_n} = \dfrac{(n^2+n)^3 - (n^3+n)^2}{\sum_{k=0}^{5} (n^2+n)^{\frac{5-k}{2}} (n^3+n)^{\frac{k}{3}}} =$

$$= \frac{\sum_{j=0}^{3} (n^2)^{3-j} n^j \binom{3}{j} - (n^6 + 2n^4 + n^2)}{-//-}$$

st $P(n) \leq 4$

$$= \frac{3n^5 + 3n^4 + n^3 - 2n^4 - n^2}{-//-} = \frac{n^5\left(3 + \frac{P(n)}{n^5}\right)}{n^5 \sum_{k=0}^{5} \left(1+\frac{1}{n}\right)^{\frac{5-k}{2}} \left(1+\frac{1}{n^2}\right)^{k/3}} \longrightarrow \frac{3}{6}$$

$$\underbrace{\sqrt{n+\sqrt{n}} - \sqrt{n+\sqrt[3]{n}}}_{b_n} = \frac{\sqrt{n} - \sqrt[3]{n}}{\sqrt{n+\sqrt{n}} + \sqrt{n+\sqrt[3]{n}}} = \frac{1 - n^{1/3-1/2}}{\sqrt{1+n^{-1/2}} + \sqrt{1+n^{1/3-1}}} \to \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{a_n}{b_n} \longrightarrow \frac{1}{2} \cdot \frac{2}{1} = 1$$

$a_n$ ... úprava ... 5

dokončení ... 3

$b_n$ ... úprava ... 4

dokončení ... 3

2. $\left((\operatorname{cotg} x)^{\operatorname{tg} x}\right)^{\frac{1}{\sin x \log x}} = e^{\frac{1}{\sin x \log x} \log (\operatorname{cotg} x)^{\operatorname{tg} x}} =$

$= e^{\frac{1}{\sin x \log x} \operatorname{tg} x \log \operatorname{cotg} x} = e^{\frac{1}{\sin x \log x} \cdot \frac{\sin x}{\cos x} \log \operatorname{cotg} x} =$

$= e^{\frac{1}{\cos x} \frac{\log \operatorname{cotg} x}{\log x}} \to e^{-1}$

$\cdot \frac{\log \operatorname{cotg} x}{\log x} = \frac{\log \cos x - \log \sin x}{\log x} = \frac{\log \cos x}{\log x} - \frac{\log \frac{\sin x}{x}}{\log x} - \frac{\log x}{\log x}$

$\xrightarrow[x \to 0_+]{} \frac{0}{-\infty} - \frac{0}{-\infty} - 1 = -1$

přepis .... 4

úprava .... 4

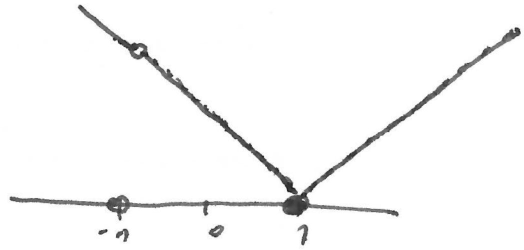
výpočet limity ... 6

závěr .... 1

3. $f(x) = (x-1)\ \mathrm{sgn}\ (x-1)(x+1)^2, \quad x \in \mathbb{R}$

$$\mathrm{sgn}\ (x-1)(x+1)^2 = \begin{cases} -1 & x \in (-\infty,-1) \\ 0 & x = -1 \\ -1 & x \in (-1,1) \\ 0 & x = 1 \\ 1 & x \in (1,\infty) \end{cases}$$

$$f(x) = \begin{cases} 1-x & \dots \ x < -1 \\ 0 & \dots \ x = -1 \\ 1-x & \dots \ x \in (-1,1) \\ 0 & \dots \ x = 1 \\ x-1 & \dots \ x > 1 \end{cases}$$




$$f'(x) = \begin{cases} -1 & \dots \ x \in (-\infty,-1) \cup (-1,1) \\ 1 & \dots \ x \in (1,\infty) \end{cases}$$

$\lim\limits_{x\to -1} f(x) = \lim\limits_{x \to -1} (1-x) = 2 \neq f(-1) \Rightarrow f$ nespojitá v $-1$

$$\left.\begin{aligned} \lim_{x\to 1_+} f(x) &= \lim_{x\to 1_+} x-1 = 0 = f(1) \\ \lim_{x\to 1_-} f(x) &= \lim_{x\to 1_-} (1-x) = 0 = f(1) \end{aligned}\right\} \Rightarrow f \text{ spojitá v } 1$$

· $f'_+(1) = \lim\limits_{x\to 1_+} f'(x) = 1 \qquad f'_-(1) = \lim\limits_{x \to 1_-} f'(x) = -1 \Rightarrow f'(1)$ neexistuje

$$\left.\begin{aligned} \cdot\ f'_+(-1) &= \lim_{x\to -1_+} \frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\to -1_+} \frac{1-x-0}{x+1} = +\infty \\ \cdot\ f'_-(1) &= \lim_{x\to -1_-} \frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\to -1_-} \frac{1-x-0}{x+1} = -\infty \end{aligned}\right\} \Rightarrow f'(-1) \text{ neexistuje}$$

$f'$ mimo $\{-1,1\}$ ... 5

$f'(-1)$ ...... 5

$f'(1)$ ...... 5

4. $f(x) = \frac{x}{2} + \operatorname{arccotg} |x+1|$

- $D(f) = \mathbb{R}$, $f$ spojitá na $\mathbb{R}$, $f(x) \xrightarrow[x\to\infty]{} \infty$, $f(x) \xrightarrow[x\to-\infty]{} -\infty$

- $f'(x) = \begin{cases} \frac{1}{2} - \frac{1}{1+(x+1)^2} & \dots\ x > -1 \\ \frac{1}{2} + \frac{1}{1+(x+1)^2} & \dots\ x < -1 \end{cases}$ $\quad f'_+(-1) = \lim_{x\to-1_+} f'(x) = -\frac{1}{2}$, $\quad f'_-(-1) = \lim_{x\to-1_-} f'(x) = \frac{3}{2}$

- $f'(x) = 0$ $\begin{cases} 1+(x+1)^2 = 2 \quad \dots\ x > -1 \\ (x+1)^2 = 1 \\ x+1 = \pm 1 \\ x = \begin{cases} 0 \\ -2 \end{cases} \Rightarrow x = 0 \\ > 0 \quad \dots\ x < -1 \end{cases}$

mon. $\nearrow$ $\searrow$ $\nearrow$

$f'$: $>0$ | $-1$ | $<0$ | $0$ | $>0$

- $H(f) = \mathbb{R}$, lokální maximum v $-1$, $0$

- $f''(x) = \begin{cases} +\frac{1}{(1+(x+1)^2)^2} 2(x+1) & \dots\ x > -1 \\ \frac{-1}{(1+(x+1)^2)^2} 2(x+1) & \dots\ x < -1 \end{cases}$

konv: konv. | $-1$ | konv.

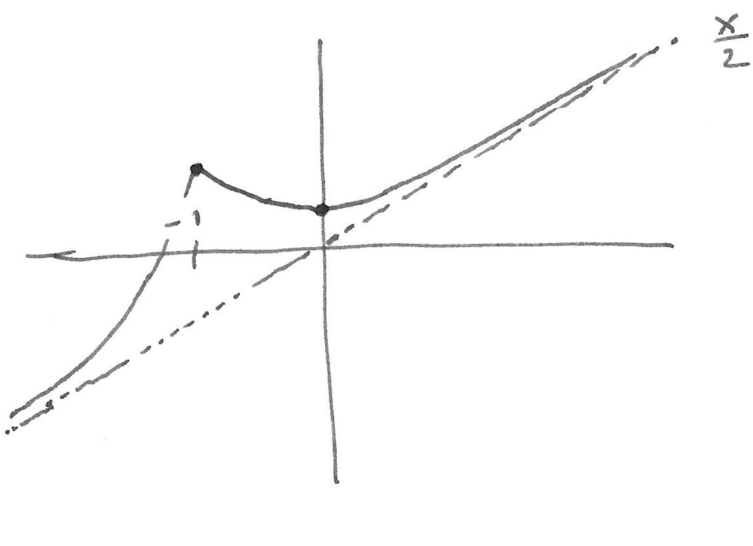
$f''$: $+$ | $-1$ | $+$

- asymptoty: $\frac{f(x)}{x} = \frac{1}{2} + \frac{1}{x} \operatorname{arccotg} |x+1| \xrightarrow[x\to\infty]{} \frac{1}{2}$

$\xrightarrow[x\to-\infty]{} \frac{1}{2}$

$+\infty$: $f(x) - \frac{x}{2} = \operatorname{arccotg} |x+1| \xrightarrow[x\to\infty]{} 0$ $\Rightarrow$ $a(x) = \frac{x}{2}$

$-\infty$: $f(x) - \frac{x}{2} = -\,/\!/\!- \xrightarrow[x\to-\infty]{} 0$ $\quad a(x) = \frac{x}{2}$




$f$ spoj .... 1

$f'$ .... 2 + 3

$f''$ .... 2 + 3

asymptoty ... 2

graf .... 2