1. $\lim\limits_{n\to\infty} (-1)^n \underbrace{\sqrt[n]{\frac{10^n+\cos n+n^3}{2^n+\sin n!+n^{10}}}}_{a_n}$

$$a_n = \frac{10\sqrt[n]{1+\frac{\cos n}{10^n}+\frac{n^3}{10^n}}}{2\sqrt[n]{1+\frac{\sin n!}{2^n}+\frac{n^{10}}{2^n}}} = 5\,\frac{\sqrt[n]{b_n}}{\sqrt[n]{c_n}}\,,$$ kde $b_n \to 1$, $c_n \to 1$

$b_n \to 1 \Rightarrow \frac{1}{2} \le b_n \le 2$ pro velké n $\Rightarrow \sqrt[n]{\frac{1}{2}} \le \sqrt[n]{b_n} \le \sqrt[n]{2}$, tedy

1

$\sqrt[n]{b_n} \to 1$. Proto $a_n \to 5$

Tedy $\left.\begin{array}{l} (-1)^{2n} a_{2n} = a_{2n} \to 5 \\ (-1)^{2n+1} a_{2n+1} = -a_{2n+1} \to -5 \end{array}\right\} \Rightarrow$ z věty o vybrané posloupnosti limita

$(-1)^n a_n$ neexistuje

vytknutí 3
$b_n, c_n \to 1$ ... 3
policajti 5

zdůvod 4

2. $$\lim_{x\to 0} \underbrace{\frac{\pi/2 - \operatorname{arctg}\frac{1}{|x|}}{\sqrt{1-\cos x}}}_{f(x)}$$

$x>0$: $$\frac{\pi/2 - \operatorname{arctg} 1/x}{\sqrt{1-\cos x}} = \frac{\pi/2 - \operatorname{arctg}\frac{1}{x}}{x}\cdot x \cdot \frac{1}{\sqrt{\frac{1-\cos x}{x^2}}\sqrt{x^2}} =$$

$$= \frac{\pi/2 - \operatorname{arctg} 1/x}{x}\cdot\frac{1}{\sqrt{\frac{1-\cos x}{x^2}}} \xrightarrow[x\to 0_+]{(*)} 1\cdot\frac{1}{\sqrt{\frac{1}{2}}} = \sqrt{2}$$

$x<0$: funkce je sudá, tedy má zleva stejnou limitu jako zprava,

proto $\lim_{x\to 0} f(x) = \sqrt{2}$ (*) $\frac{\pi/2 - \operatorname{arctg} 1/x}{x} \xrightarrow[x\to 0_+]{} 1$ dle l'H

arctg ... 6

cos 4

absolutní hodnota ... 4

závěr 1

3. $f(x) = \begin{cases} \dfrac{x - \sin x}{x^3} & \dots\ x \neq 0 \\ 1/6 & \dots\ x = 0 \end{cases}$

pak $\lim\limits_{x \to 0} f(x) \overset{l'H}{=} \lim \dfrac{1 - \cos x}{3x^2} = \dfrac{1}{6} = f(0)$, tedy f spojitá na $\mathbb{R}$

$x \neq 0$, pak $f'(x) = \dfrac{1}{x^6}\left((1 - \cos x)\,x^3 - (x - \sin x)\,3x^2\right)$

Dále $f'(0) = \lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0} \dfrac{\frac{x - \sin x}{x^3} - 1/6}{x} =$

$= \lim\limits_{x \to 0} \dfrac{6x - 6\sin x - x^3}{6x^4} \overset{l'H}{=} \lim\limits_{x \to 0} \dfrac{6 - 6\cos x - 3x^2}{24x^3} \overset{l'H}{=} \lim\limits_{x \to 0} \dfrac{6\sin x - 6x}{72x^2} =$

$= \dfrac{6}{72} \lim\limits_{x \to 0} \dfrac{\sin x - x}{x^3}\, x = \dfrac{6}{72} \cdot \dfrac{-1}{6} \cdot 0 = 0$

spojitost na $\mathbb{R}$ 4

derivace na $\mathbb{R} \setminus \{0\}$ ... 4

derivace v 0 7

4. $f(x) = \sin x + \dfrac{1}{6\sin x}$, $x \in (0, \pi)$

- f spojitá na $(0,\pi)$
- $\lim\limits_{x\to 0_+} f(x) = \infty$, $\lim\limits_{x\to \pi_-} f(x) = \infty$
- $f'(x) = \cos x + \dfrac{-\cos x}{6\sin^2 x} = \cos x - \dfrac{1}{6}\dfrac{\cos x}{\sin^2 x} = \cos x\left(1 - \dfrac{1}{6\sin^2 x}\right)$

$f'(x) = 0 \iff x_1 = \pi/2$ nebo $\sin x = \pm \dfrac{1}{\sqrt{6}}$ ($\sin > 0$ na $(0,\pi)$), tedy

$\sin x = 1/\sqrt{6}$, tj. $x_2 = \arcsin \dfrac{1}{\sqrt{6}}$

nebo $x_3 = \pi - \arcsin \dfrac{1}{\sqrt{6}}$

f		$\searrow$		$\nearrow$		$\searrow$		$\nearrow$	
f'		$-$		$+$		$-$		$+$	
	0		x_2		$\pi/2 = x_1$		x_3		π

Tedy v x_1 lokální maximum, v x_2, x_3 globální minimum

$\left(f(x_2) = f(x_3) = \dfrac{1}{\sqrt{6}} + \dfrac{1}{6\frac{1}{\sqrt{6}}} = \dfrac{1}{\sqrt{6}} + \dfrac{1}{\sqrt{6}} = \dfrac{2}{\sqrt{6}} \right)$

Tedy $H_f = \left\langle \dfrac{2}{\sqrt{6}}, \infty \right)$

- $f''(x) = -\sin x - \dfrac{1}{6}\dfrac{-\cancel{\sin x}\sin^2 x - \cos x\, 2\cancel{\sin x}\cos x}{\sin^{\cancel{4}3} x} =$

$= -\sin x - \dfrac{1}{6}\dfrac{-\sin^2 x - 2\cos^2 x}{\sin^3 x}$

pak $f''(x) = 0 \iff -6\sin^4 x - \sin^2 x + 2 = 0$

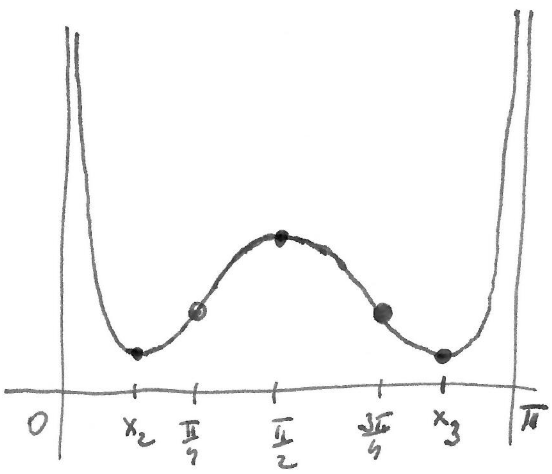
$\sin^2 x = y \Rightarrow -6y^2 - y^2 + 2 = 0$

$y_{1,2} = \dfrac{1 \pm \sqrt{49}}{-12} = \begin{cases} -\frac{2}{3} \\ 1/2 \end{cases}$

$\Rightarrow \sin x = \dfrac{1}{\sqrt{2}} \Rightarrow x = \pi/4, \dfrac{3\pi}{4}$

	konv.		konk.		konv.
	$+$		$-$		$+$
0		$\frac{\pi}{4}$		$\frac{3\pi}{4}$	

inflexe



spojitost ... 1

1. derivace ... 7 = 3 + 2 + 2

2. derivace ... 7 = 3 + 2 + 2

obrázek ... 0