Selected topics in harmonic analysis 1 – WS 2023 - 2024
Information about the exam

The exam will be oral and can be held either in English or in Czech, depending on the preference of the student. Please contact me in order to schedule a time for your exam.

The exam will consist of three questions:

(i) one definition; (ii) one theorem without proof; (iii) one theorem with proof.

A list of possible topics for each of these questions is listed below. For question (iii), proofs will be required only to the extent in which they were presented in the lectures and emphasis will be put on the understanding of the basic idea of the proof, rather than on technical details.

(i)

Space $\mathcal{M}_p(\mathbb{R}^n)$
Hilbert transform
Riesz transforms
Homogeneous singular integral operator
Calderón-Zygmund singular integral operator

(ii)

Characterization of translation-invariant operators (Theorem 1)
Characterization of the space $\mathcal{M}_1(\mathbb{R}^n)$ (Theorem 2)
Characterization of the space $\mathcal{M}_2(\mathbb{R}^n)$ (Theorem 3)
Van der Corput lemma (Theorem 4)
Hilbert transform as a Fourier multiplier operator (Theorem 5)
L^2 boundedness of homogeneous singular integral operators (Theorem 6)
L^2 boundedness of Calderón-Zygmund singular integral operators (Theorem 7)
Calderón-Zygmund decomposition (Theorem 8)
L^2 boundedness implies L^p boundedness (Theorem 9)
L^p boundedness of odd homogeneous singular integral operators (Theorem 10)

(iii)

Characterization of the space $\mathcal{M}_2(\mathbb{R}^n)$ (Theorem 3)
Van der Corput lemma (Theorem 4)
Hilbert transform as a Fourier multiplier operator (Theorem 5)
L^2 boundedness of homogeneous singular integral operators (Theorem 6)
Calderón-Zygmund decomposition (Theorem 8)
L^2 boundedness implies L^p boundedness (Theorem 9)
L^p boundedness of odd homogeneous singular integral operators (Theorem 10)