Existence and exact multiplicity of positive periodic solutions to undamped parameter-dependent second-order super-linear ODEs

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We shall discuss the existence and exact multiplicity of positive periodic solutions to the parameter-dependent problem

$$u'' = p(t)u - q(t,u) + \mu f(t); \quad u(0) = u(\omega),\ u'(0) = u'(\omega), \tag{1}$$

where $q\colon [0,\omega] \times [0,+\infty[\to \mathbb{R}$ has super-linear growth at infinity and μ is a real parameter.

Since the main hypotheses on q are motivated by properties of the power non-linearities, for the sake of simple and clear formulations, we shall discuss a particular case of (1), namely, the problem

$$u'' = p(t)u - h(t)|u|^{\lambda} \operatorname{sgn} u + \mu f(t); \quad u(0) = u(\omega),\ u'(0) = u'(\omega) \tag{2}$$

in which $\lambda > 1$. We shall show, among other things, that if we consider μ as a bifurcation parameter, crossing a certain critical value μ_0, a bifurcation of positive solutions to problem (2) occurs.

The proofs of our results are based on the method of lower and upper functions, combined with the technique of differential inequalities (maximum and anti-maximum principles).