Positive Periodic Solutions in Population Models

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Quite wide class of population models leads to a functional differential equation

$$u'(t) = \ell(u)(t) + \lambda F(u)(t) \quad \text{for a. e. } t \in \mathbb{R}, \tag{1}$$

where ℓ and $F : C_\omega(\mathbb{R}) \to L_\omega(\mathbb{R})$ are, respectively, linear and nonlinear operators transforming the space of continuous ω-periodic functions into the space of locally Lebesgue integrable ω-periodic functions, and $\lambda \in \mathbb{R}$ is a real parameter. We are interested in the existence of a positive ω-periodic solution to (1) particularly in the case when $F(0)(t) = 0$ for a. e. $t \in \mathbb{R}$. We will establish conditions guaranteeing the existence of a bifurcation point λ_c for ω-periodic solutions to the equation (1) provided the Green's function to the corresponding linear periodic boundary value problem is positive. The question how to estimate λ_c numerically will also be discussed.