Multiplicity of periodic solutions for asymptotically linear planar systems

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We discuss the existence and multiplicity of T-periodic solutions for a T-periodic planar system $\dot{x} = g(t,x)$ asymptotically linear in zero and at infinity, i.e. $g(t,0) = 0$, $D_x g(t,0) = \mathbb{A}_0(t)$ and

$$\lim_{|x|\to+\infty} \frac{|g(t,x) - \mathbb{A}_\infty(t)x|}{|x|} = 0 \quad \text{uniformly in } t \in [0,T]$$

assuming that the two linear systems with matrices $\mathbb{A}_0$ and $\mathbb{A}_\infty$ have sufficiently different rotational properties.

If the system has an Hamiltonian structure, couples of nontrivial periodic solutions with a specific rotation number can be found by applying the Poincaré–Birkhoff theorem. In this talk, we combine this classical approach with a novel method incorporating Brouwer's degree and rotational properties, to find additional periodic solutions. A sharp lower bound on the number of periodic solutions is obtained, expressed in terms of Maslov's indices of the linearized systems at zero and infinity.

References

[1] P. Gidoni and A. Margheri, Lower bound on the number of periodic solutions for asymptotically linear planar Hamiltonian systems, Discrete & Continuous Dynamical Systems–A 39 (2019), 585–605.

[2] P. Gidoni, A topological degree theory for rotating solutions of planar systems, preprint arXiv:2108.13722.