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## Associative quasistatic delamination model based on energetic solutions and its BEM implementation

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### Abstract

The problem of quasistatic rate-independent evolution for problems of elastic-brittle delamination is considered. Delamination processes for linear elastic bodies glued by an adhesive to each other or to a rigid outer boundary are studied. The energy amounts dissipated in fracture Mode I (opening) and Mode II (shearing) at the interfaces are distinguished [1, 2], an inelastic process at these interfaces being devised in such a way that the dissipated energy depends only on the rates of internal parameters and therefore the model is associative.

An internal variable having the meaning of the plastic tangential slip along the interface is introduced and an elastoplastic-type model with kinematic-type hardening devised. Additionally to an interface plasticity, a damage along the interfaces is taken into account introducing a damage variable [3] which may take values in the range between unity and zero, with the value of unity standing for the non damaged state while that of zero for the totally damaged state. Solutions are defined within the context of the so-called energetic solutions [4, 5, 6].

After introducing a time discretization pattern a boundary discretization is considered and the boundary element method [7] is utilized to solve the respective boundary value problems for sub-domains and compute the stored elastic energy.

A few sample problems, solved by an in-house code [8], are studied in order to illustrate the capabilities of the numerical method developed.

## ***References***

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