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Rate-independent systems. Theory and application. (English) Zbl 1339.35006
Applied Mathematical Sciences 193. New York, NY: Springer (ISBN 978-1-4939-2705-0/hbk; 978-1-4939-2706-7/ebook). xxii, 660 p. (2015).

The book focuses on the study of quasi-static evolutionary systems which are rate-independent and gives examples of such situations, mainly in continuum mechanics. It is divided in five chapters completed with a short introduction which gives a an historical presentation of the material and with three appendices.

In the short Chapter 1, the authors present the basic notions of evolutionary systems. They start with the ode $M \ddot{\to} q + F(\dot{\to} q) + Kq = \widehat{l}(t)$, which is a balance equation which describes the evolutions of the state q with respect to the time parameter, under the action of the external force $\widehat{l}$. M and K are symmetric and positive-definite matrices and F is the possibly nonlinear damping. The authors exhibit three time scales respectively associated to $M^{-1}K$, $V^{-1}K$ (assuming $F(q) = Vq$ for a symmetric and positive-definite matrix V) and $\inf \left\| \frac{d}{dt}\widehat{l}(t) \right\|$. They define the notion of rate-independent system when the two first time scales are much faster than the third one. They give an example considering $\widehat{l}(t) = l(\varepsilon t)$ with $\varepsilon > 0$ and $F(\dot{\to} q) = \nu \left|\dot{\to} q\right|^{\alpha-1} \dot{\to} q$. They indeed obtain the final ode $\varepsilon^2 M \ddot{\to} q + \varepsilon^\alpha \nu \left|\dot{\to} q\right|^{\alpha-1} \dot{\to} q + Kq = l(t)$, leading to the inclusion problem $\partial\mathcal{R}(\dot{\to} q) + Kq \ni l(t)$, where $\mathcal{R}(\dot{\to} q) = \nu \left|\dot{\to} q\right|^\alpha$ is the dissipation potential, when taking the limit when ε tends to 0. This last problem may be generalized as Biot's equation $\partial\mathcal{R}(\dot{\to} q) + D_q\mathcal{E}(t,q) \ni 0$, introducing $\mathcal{E}(t,q) = \frac{1}{2}\langle q, Kq\rangle - \langle l(t), q\rangle$, D_q being the partial differential operator with respect to q. The authors then give the definition of a rate-independent system, among other definitions: admissible, concatenation property, restriction property and causal systems. They give examples of such systems. They also define the notion of general evolutionary system as $\partial_{\dot{\to} q}\mathcal{R}(q, \dot{\to} q) + D_q\mathcal{E}(t,q) \ni 0$, starting from an initial state q_1 at time t_1. Here $\mathcal{R} : \mathbf{Q} \to [0,\infty]$ is convex and lower semicontinuous satisfying $\mathcal{R}(0) = 0$ and $\partial_{\dot{\to} q}$ denotes the subdifferential operator with respect to $\dot{\to} q$ and $D_q\mathcal{E}$ means the Gateaux derivative of $\mathcal{E}$ with respect to $q \in \mathbf{Q}$, the Banach state space. In order to obtain a rate-independent system, the authors have to impose a p-homogeneous property on the dissipation potential $\mathcal{R}$: $\mathcal{R}(q,\cdot)$ is p-homogeneous for every $q \in Q$. This may be generalized splitting the state variable into its nondissipative and dissipative components and assuming that the dissipation potential only depends on the dissipative component. The authors then derive some general properties of such problems among which are a priori estimates on the state q. They define the notion of energetic solution, they prove the existence of an energetic solution under hypotheses on the initial condition and they analyze some examples. They also build an approximation scheme for such an energetic solution. They also introduce other notions of solutions for special problems and they compare these notions in the 1D case.

In Chapter 2, the authors analyze energetic rate-independent systems. They here consider a state space $\mathcal{Q}$ which splits as $\mathcal{Q} = \mathcal{Y} \times \mathcal{Z}$. After the general definition of such systems and of dissipative distance, from which they deduce that of energetic solution, they prove several existence results in different realistic situations. The chapter ends with an application of Γ-convergence tools in the present context in order to obtain relaxation results.

In Chapter 3, the authors assume that the state space $\mathbf{Q}$ splits as $\mathbf{Q} = \mathbf{Y} \times \mathbf{Z}$, where $\mathbf{Y}$ and $\mathbf{Z}$ are separate and reflexive Banach spaces. The purpose of this chapter is to define the appropriate notions of rate-independent system and of solution within this context. The authors start with the notion of energetic solution and they prove an existence result under hypotheses on the initial condition. In order to define more general solutions, they recall the notions of monotone, maximal monotone, responsive and maximal responsive operators $A : \mathbf{Z} \rightrightarrows \mathbf{Z}^*$ and they establish properties of such operators. After the presentation of the notions of convex, Fréchet, limiting, Clarke and Dini subdifferentials, the authors introduce the notions of differential, semi-differential, continuous dissipative (CD), local and weak solution for evolutionary systems. They establish some links between these notions and an existence result for an energetic solution which is also a CD solution. The case of convex systems, that is such that $\mathcal{R}$ is translation-invariant and $\mathcal{E}(t,.)$ is convex, is considered. The authors assume that $\mathcal{Q}$ is a convex subset of the Banach state space $\mathbf{Q}$. They prove that energetic solutions may be generated by a.e. local solutions and every energetic solution is proved to be continuous with respect to the time parameter, under some hypotheses. Then every energetic solution may be a differential or a semi-differential solution under appropriate hypotheses. The authors also prove some uniqueness results for energetic solutions of rate-independent and convex systems. The next section of this chapter focuses on the special case where $\mathcal{E}(t,q) = \frac{1}{2}\langle q, Kq\rangle - \langle l(t), q\rangle$, with $l \in W^{1,1}(0,T;\mathbf{Q}^*)$ and $A : \mathbf{Q} \to \mathbf{Q}^*$ is a symmetric, positive semi-definite, bounded and linear operator. When A is positive definite, the authors prove an existence and uniqueness result for an energetic solution $q \in W^{1,1}(0,T;\mathbf{Q})$ starting from an initial condition $q_0 \in A^{-1}(l(0) - \Sigma)$. When A is only positive semi-definite, they prove an existence result under further hypotheses on $\mathbf{Q}$. This section ends with applications of Γ -convergence tools and homogenization results. The next section describes some numerical tools for the computation of an energetic solution and the authors prove stability and convergence results. The chapter ends with two further considerations on rate-independent systems in this context.

The very long Chapter 4 moves to applications of the notions and tools which have been presented in the previous chapters in continuum mechanics and physics of solids. The chapter starts with a detailed presentation of continuum mechanics. They recall the notions of stress, strain, energy in the case of small or large strains, then that of elasticity and plasticity. They prove an existence result for an energetic solution in the polyconvexity case under hypotheses on the energy and they propose a numerical scheme for some examples. A long section is then devoted to the study of inelastic processes at small strains. The authors again prove an existence results for a unique solution which belongs to $W^{1,\infty}(0,T;H^1(\Omega,\mathbb{R}^d))$, and they prove the strong convergence of the associated numerical scheme. They present several examples. The chapter ends with a section dedicated to the study of ferromagnetic materials. Once again, the authors prove an existence result and the convergence of the numerical scheme.

In the final Chapter 5, the authors analyze different realistic examples involving evolutionary systems. They start with the case of a hyperviscoelastic material with the Kelvin-Voigt rheology leading to the momentum equilibrium $\rho \dashrightarrow u - \mathrm{div}\sigma = f$, with $\sigma(e,z) = W'_e(e,z) + R'_{\dashrightarrow e}(e,z,\dashrightarrow e)$, e being the linearized strain tensor. Here R is a pseudopotential of viscous like dissipative forces. The authors put this problem in an abstract way $M \dashrightarrow u + V(u,z) \dashrightarrow u + \partial_u \mathcal{E}(t,u,z) = f(t,u,z)$, and they add initial conditions for u, $\dashrightarrow u$ and z. They prove an existence result for local and energetic solutions and an approximation result by time discretization. They then prove an existence result for a problem written in an abstract setting and the chapter ends with applications in plasticity, damages in a viscoelastic material, delaminations of viscoelastic bodies...

Two appendices complete this nice and detailed presentation. The first one recalls useful notions of topology and of functional analysis, while the second one is dedicated to elements of measure theory and of function spaces. The third appendix presents the basis of Young measures and their properties.

The book is written by two famous specialists of evolutionary systems as they worked for a long time on this topic. Throughout the whole book, the authors give lots of precise details concerning the particularities of the rate-independent systems in different and realistic settings. They take care to illustrate their theoretical notions with many examples and they complete their computations with many figures.

Reviewer: Alain Brillard (Riedisheim)

MSC:

35-02	Research monographs (partial differential equations)	Cited in **4** Documents
35K85	Linear parabolic unilateral problems; linear parabolic variational inequalities	
35Q74	PDEs in connection with mechanics of deformable solids	
47J35	Nonlinear evolution equations	
49J40	Variational methods including variational inequalities	
49J45	Optimal control problems involving semicontinuity and convergence; relaxation	
49S05	Variational principles of physics	
74-02	Research monographs (mechanics of deformable solids)	
74C15	Large-strain, rate-independent theories	
35B27	Homogenization; equations in media with periodic structure (PDE)	

Keywords:

dissipative potential; energetic solution; subdifferential; convex system; Γ-convergence; numerical scheme; damage; delamination; Young measures; quasi-static evolutionary systems

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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