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★**Rate-independent systems.**
Theory and application.
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In materials science time is a fundamental variable but usually the best we can do is to consider equilibrium problems and this is the standard if we are considering nonlinear constitutive equations. To have an idea of the complexity of time dependent phenomena it is sufficient to remember that the theory of elasticity is concerned with statics for all simple materials. Therefore, as soon we try to understand time dependent phenomena a *mare magnum* just opens. Constitutive equations that are rate-independent, i.e. invariant under a change of time scale, are a fundamental class of mathematical models for investigating time dependence in materials science beyond inertia and viscous forces. The first authors to provide a deep reflection on the concept of rate-independence in the framework of the modern theory of continuum mechanics were A. C. Pipkin and R. S. Rivlin [Z. Angew. Math. Phys. **16** (1965), 313–327; MR0203992] and D. R. Owen and W. O. Williams ["On the concept of rate-independence", tech. rep., Carnegie Mellon Univ., 1967] in the sixties, but a systematic and organized reflection on this topic is still missing and for this reason the present book is surely welcomed.

Rate-independent systems: theory and application is a very *rich* book: 660 pages and more than 600 references. The book is divided into five chapters and three appendices. There is no name index. The first chapter is an introduction and at the same time a survey of all the topics touched upon in the book. Chapters 2 and 3 are about the mathematics of rate-independent systems. Chapter 4 is about applications in the mechanics and the physics of solids. Chapter 5 is an invitation to go on considering *some* rate dependence.

The authors declare that they expect the reader to be familiar with several advanced topics in the fields of functional analysis, calculus of variations, and partial differential equations. They suggest the book by P. G. Ciarlet [*Mathematical elasticity. Vol. I*, Stud. Math. Appl., 20, North-Holland, Amsterdam, 1988; MR0936420] as a good introduction for Chapter 4 and the book [T. Roubíček, *Nonlinear partial differential equations with applications*, second edition, Internat. Ser. Numer. Math., 153, Birkhäuser/Springer Basel AG, Basel, 2013; MR3014456] for Chapter 5. The appendices are devoted to mathematical topics.

One of the main points of the book is to develop a mathematical machinery that is able to work with nonlinear and nonconvex constitutive equations. By considering an abstract definition of rate-independent systems obtained using the notion of input-output systems of control theory, the authors are able to study a huge class of quasistatic evolutionary systems ranging from damage to ferromagnetism. This gives an encyclopaedic value to the book. Only Chapter 5 is restricted to small strains.

Chapters 2 and 3 are very interesting. They give a deep overview of a list of mathematical methods that are very useful in studying the mathematical behavior of different models. To the best of my knowledge, only in this book is it possible to find a detailed treatment of all these methods in the same place. Starting from energetic solutions we arrive at differential solutions via several methods such as Γ-convergence, Young's measures, BV solutions, and vanishing viscosity.

In my opinion, Chapter 4 (more than 200 pages) is mainly of encyclopaedic interest. Elasticity, plasticity, phase transformation in smart materials, debonding, cracks in brittle materials, damage, magnetostriction and piezoelectricty are some of the problems considered. Clearly it is not possible to develop all this in detail, but it is possible to give interesting information at least on some aspects. This is what happens in the case of nonlinear elasticity. In only 12 pages it is not possible to truly understand the subject but we can have a little survey illustrating how the mathematics of elastic solids is related to the mathematical methods considered in the previous chapters. Sometimes information about numerical problems is given.

Chapter 5 is again about applications but it is different in nature and scope from the previous one. Indeed, here we can find more details on how the theory developed in Chapters 2 and 3 can be applied beyond rate-independent materials. It is clear that much of what this chapter describes could be considered work in progress. Therefore, this chapter might be an interesting starting point for new research.

The book is an interesting and useful piece of applied mathematics. It is of definite interest for researchers involved in the mathematical issues of the models in materials sciences. The book contains a clear treatment of the mathematical methods we need and a survey of many applications. It would be nice if in the future a companion book about the mechanics of rate-independent models were published. The mathematical methods that we master today are so powerful that it is time to reflect on our mechanical understanding of rate-independence in a more systematic way. *Giuseppe Saccomandi*