1. $11 / 3^{66} - 8$

$n / a - b \iff a \equiv b \mod n$

$a = 3^{66}$
$b = 8$

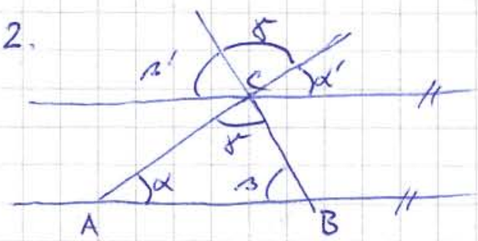
$$\left.\begin{array}{l}
3^{66} = 27^{22} = (22+5)^{22} = p + 5^{22}; \quad p \bmod 11 = 0 \Rightarrow 3^{66} \equiv 5^{22} \mod 11 \\
5^{22} = 25^{11} = (22+3)^{11} = q + 3^{11}; \quad q \bmod 11 = 0 \Rightarrow 5^{22} \equiv 3^{11} \mod 11 \\
3^{11} = 3^2 \cdot 3^9 = 9 \cdot 27^3 = 9 \cdot (22+5)^3 = 9 \cdot r + 9 \cdot 5^3 \ ; \ 9r \bmod 11 = 0 \Rightarrow 3^{11} \equiv 9 \cdot 5^3 \mod 11 \\
9 \cdot 5^3 = 9 \cdot 5 \cdot 25 = 9 \cdot 5 \cdot (22+3) = t + 45 \cdot 3 \ ; \ t \bmod 11 = 0 \Rightarrow 9 \cdot 5^3 \equiv 45 \cdot 3 \mod 11 \\
45 \cdot 3 = (44+1) \cdot 3 = 44 \cdot 3 + 3 = u + 3 \ ; \ u \bmod 11 = 0 \Rightarrow 45 \cdot 3 \equiv 3 \mod 11
\end{array}\right] (*)$$

1b

$(*) \Rightarrow 3^{66} \equiv 3 \mod 11$

$3 \not\equiv 8 \mod 11 \Rightarrow 3^{66} \not\equiv 8 \mod 11 \Rightarrow$ $\underline{11 \text{ nedělí } 3^{66} - 8}$

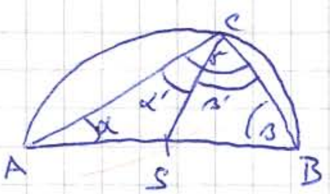
2.



$(\alpha' = \alpha) \wedge (\beta' = \beta) \wedge ((\alpha' + \beta' + \gamma) = 180°) \Rightarrow \underline{\underline{\alpha + \beta + \gamma = 180°}}$

$(**)$ 1b

3.



$|SB| = |SC| \Rightarrow \beta' = \beta$
$|SA| = |SC| \Rightarrow \alpha' = \alpha$

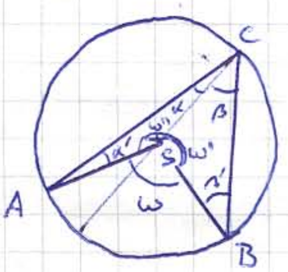
$\gamma = \alpha' + \beta' = \alpha + \beta$

$\alpha + \beta + \gamma = 180°$ z $(**)$

$\alpha + \beta + \gamma = \alpha + \beta + \alpha' + \beta' = 2\alpha + 2\beta = 180° \Rightarrow \alpha + \beta = 90° \Rightarrow$ 1b

$\Rightarrow \underline{\gamma = 90°}$

4.



$|AS| = |SC| \Rightarrow \alpha = \alpha'$
$|BC| = |BS| \Rightarrow \beta = \beta'$

$\alpha + \alpha' + \omega'' = 180° = 2\alpha + \omega''$ (1)
$\beta + \beta' + \omega' = 180° = 2\beta + \omega'$ (2)

$\omega + \omega' + \omega'' = 360° \Rightarrow \omega = 360° - (\omega' + \omega'')$

1b

$(1) + (2): 360° = 2 \cdot (\alpha + \beta) + \omega'' + \omega' \quad / - (\omega'' + \omega')$

$\underbrace{360° - (\omega' + \omega'')}_{\omega} = 2(\alpha + \beta)$

$\omega = 2(\alpha + \beta)$

5.

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\
1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \\
1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1 \\
1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1 \\
1 \quad 11 \quad 55 \quad 165 \quad 330 \quad 462 \quad 462 \quad 330 \quad 165 \quad 55 \quad 11 \quad 1
\end{array}
$$

$11 \mid n^{11} - n$

$n = 1$

$n^{11} - n = 0 \quad 11 \mid 0 \quad$ I.P. ✓

$$(n+1)^{11} - (n+1) = \underline{n^{11}} + 11n^{10} + 55n^9 + 165n^8 + 330n^7 + 462n^6 + 462n^5 + 330n^4 + 165n^3 + 55n^2 + 11n + 1 \underline{-n} - 1 =$$

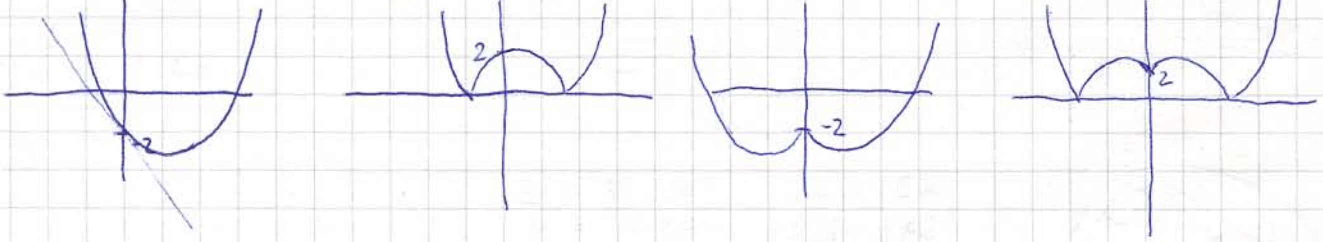
$$= \left| \begin{array}{ll} 11 \bmod 11 = 0 & 330 \bmod 11 = 0 \\ 55 \bmod 11 = 0 & 462 \bmod 11 = 0 \\ 165 \bmod 11 = 0 & \end{array} \right| = q + n^{11} - n$$

$q \bmod 11 = 0 \quad \wedge \quad$ předpoklad byl $(n^{11} - n) \bmod 11 = 0$

$\Rightarrow 11 \mid n^{11} - n$

1b

6. $f(x) = 2x^2 - 3x - 2$ $\qquad f(x) = |2x^2 - 3x - 2|$ $\qquad f(x) = 2x^2 - 3|x| - 2$ $\qquad f(x) = |2x^2 - 3|x| - 2|$



4b

7. $f(1) = -2 \quad f(2) = 4 \quad f(3) = 4$

$$
\begin{array}{lr}
a + b + c = -2 & (1) \\
4a + 2b + c = 4 & (2) \\
9a + 3b + c = 4 & (3)
\end{array}
$$

$(2)-(1): \quad 3a + b = 6 \qquad (4)$

$(3)-(1): \quad 8a + 2b = 6 \Leftrightarrow 4a + b = 3 \qquad (5)$

$(4)-(5): \quad \underline{\underline{a = -3}}$

z (4): $3(-3) + b = 6$

$\underline{\underline{b = 15}}$

z (1)

$-3 + 15 + c = -2$

$\underline{\underline{c = -14}}$

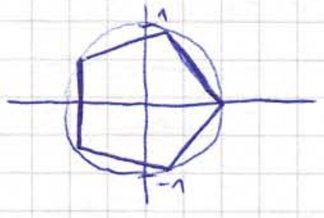
$f(x): \underline{-3x^2 + 15x - 14 = 0}$

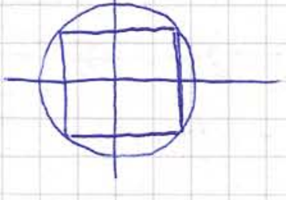
1b

8. $i^{175} = i^{4 \cdot 43 + 3} = i^3 = -i$

1b

9. $z^5 = 1 \qquad\qquad z^4 = -1$

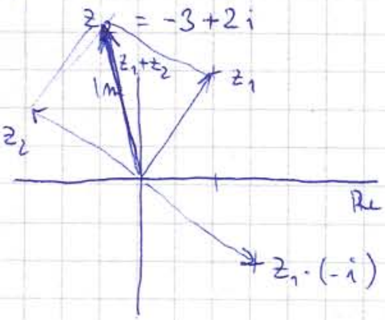




2b

10. $z_1 = 2+3i$

$z_2 = -3+2i$



4b

11.

$z_1 \cdot z = 0 + yi \qquad y \in \mathbb{R} \quad , z = a + bi$

$(2+3i)\cdot(a+bi) = 2a + 2bi + 3ai - 3b = \underbrace{(2a-3b)}_{Re} + \underbrace{(2b+3a)}_{Im} i$

$2a - 3b = 0$

$2a = 3b \qquad b \in \mathbb{R}$

$a = \frac{3}{2} b \qquad a = \frac{2}{3} b$

1b

$(2+3i)\cdot(\frac{2}{3}b + bi) = yi \qquad b \in \mathbb{R}$

12.

$z = (7+2i)^{32} = (\sqrt{53})^{32} (\cos 32\alpha + i \sin 32\alpha) = (*)$

$7+2i = \sqrt{53} (\cos\alpha + i \sin\alpha) =$

$\cos\alpha = \frac{7}{\sqrt{53}} \qquad \operatorname{tg}\alpha = \frac{2}{7} \Rightarrow \alpha \doteq 15,94°$

$\sin\alpha = \frac{2}{\sqrt{53}}$

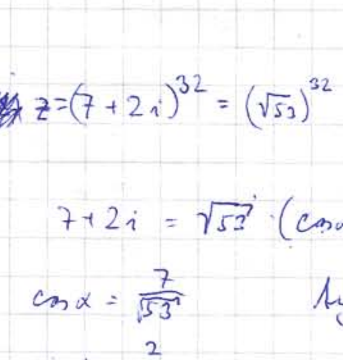
$(*) = 53^{16} \cdot (\cos 32 \cdot 15,94° + i \sin 32 \cdot 15,94°) = 53^{16} \cdot (\cos 150,25 + i \sin 150,25)$

$|z| = 53^{16}$

argument $: 150,25°$

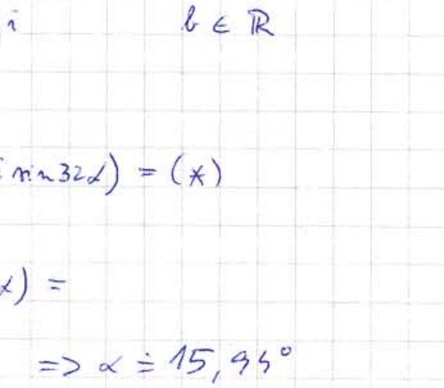
1b

13. $|z - a - bi| = c \qquad |z - 2 - i| > 4 \qquad 1 < |z + 3i - 2| \leq 4$



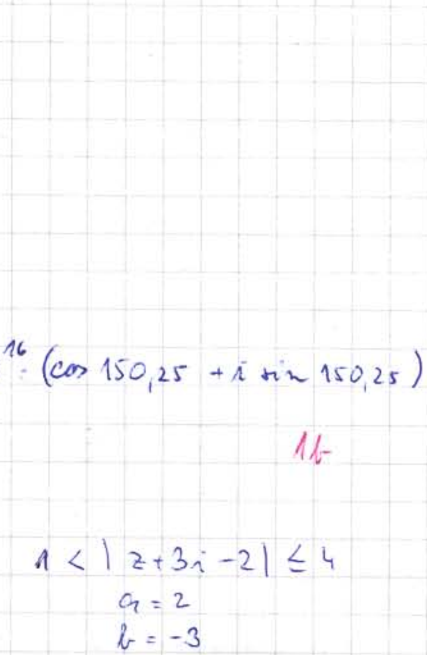
$a = 2$

$b = 1$



$a = 2$

$b = -3$



2b

14. • $z = -7-7i$

$|z| = \sqrt{49+49} = 7\sqrt{2}$

$z = -7-7i = 7\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = 7\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$ cos<0 sin<0 III kv

$\alpha = 225^\circ$
$z = 7\sqrt{2}\,(\cos 225^\circ + i\sin 225^\circ)$

• $z = 1 + \cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi$

$|z| = \sqrt{(1+\cos\frac{3}{4}\pi)^2 + \sin^2\frac{3}{4}\pi} = \sqrt{1+2\cos\frac{3}{4}\pi + \cos^2\frac{3}{4}\pi + \sin^2\frac{3}{4}\pi} = \sqrt{2+2\cos\frac{3}{4}\pi} = \sqrt{2-2\frac{\sqrt{2}}{2}} = \sqrt{2-\sqrt{2}}$

$\cos\alpha = \frac{1+\cos\frac{3}{4}\pi}{\sqrt{2-\sqrt{2}}}$

$\sin\alpha = \frac{\sin\frac{3}{4}\pi}{\sqrt{2-\sqrt{2}}}$

$\operatorname{tg}\alpha = \frac{\sin\frac{3}{4}\pi}{1+\cos\frac{3}{4}\pi} \Rightarrow \alpha = \operatorname{arctg}\left(\frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}\right) = \operatorname{arctg}\frac{\sqrt{2}}{2-\sqrt{2}} \doteq 67{,}5^\circ$

•
$z = -\sqrt{3} + i$
$|z| = \sqrt{4} = 2$

$\left.\begin{array}{ll}\cos\alpha = \frac{-\sqrt{3}}{2} & <0 \\ \sin\alpha = \frac{1}{2} & >0\end{array}\right] \Rightarrow$ II. kvadrant $\Rightarrow \alpha = 150^\circ$

• $z = \sin 30^\circ + i\cos 30^\circ = 1\cdot(\cos 60^\circ + i\sin 60^\circ)$

$\sin 30^\circ = \cos 60^\circ$
$\cos 30^\circ = \sin 60^\circ$

4b

15 • $z^4 = 3+2i$

$z^4 = |z|^4(\cos 4x + i\sin 4x) = |a|(\cos\alpha + i\sin\alpha) = 3+2i = \sqrt{13}\left(\frac{3}{\sqrt{13}} + \frac{2}{\sqrt{13}}i\right)$

$|a| = \sqrt{9+4} = \sqrt{13}$

$\operatorname{tg}\alpha = \frac{2}{3} \Rightarrow \alpha \doteq 33{,}69^\circ + k\cdot 360^\circ \Rightarrow x_1 = \frac{33{,}69^\circ}{4} \doteq 8{,}42^\circ$

$x_2 = \frac{33{,}69+360}{4} \doteq 98{,}42^\circ$

$x_3 \doteq 188{,}42^\circ$

$x_4 = 278{,}42^\circ$

$|z|^4 = \sqrt{13} \Rightarrow z = \sqrt[8]{13}$

$z_1 = \sqrt[8]{13}\,(\cos 8{,}42^\circ + i\sin 8{,}42^\circ)$
$z_2 = \sqrt[8]{13}\,(\cos 98{,}42^\circ + i\sin 98{,}42^\circ)$
$z_3 = \sqrt[8]{13}\,(\cos 188{,}42^\circ + i\sin 188{,}42^\circ)$
$z_4 = \sqrt[8]{13}\,(\cos 278{,}42^\circ + i\sin 278{,}42^\circ)$

- $z = \sqrt[7]{128}$

$|z|^7 = 128 \Rightarrow |z| = \sqrt[7]{128} = 2$

$z^7 = |z|^7(\cos 7x + i\sin 7x) = 128\cdot(\cos\alpha + i\sin\alpha) = 128\cdot(1 + 0i)$

$\alpha = 0° + k\cdot 360°$

$x_1 = \frac{0}{7} = 0$
$x_2 = \frac{360}{7} \doteq 51{,}42°$
$x_3 = \frac{2\cdot 360}{7} \doteq 102{,}85$
$x_4 = \quad \doteq 154{,}28$
$x_5 = \quad \doteq 205{,}71$
$x_6 = \quad \doteq 257{,}14$
$x_7 = \quad \doteq 308{,}57$

$\underline{z_j = 2\cdot(\cos x_j + i\sin x_j)}$

- $z = \sqrt[3]{-2-2i}$

$z^3 = |z|^3(\cos 3x + i\sin 3x) = -2-2i = |a|\cdot(\cos\alpha + i\sin\alpha) = 2\sqrt{2}\left(\frac{-2}{2\sqrt{2}} - \frac{2}{2\sqrt{2}}i\right) = 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

$|a| = \sqrt{8} = 2\sqrt{2}$ $\qquad \alpha = 225° + k\,360°$

$|z|^3 = \sqrt{8} \Rightarrow |z| = \sqrt[6]{8}$

$x_1 = \frac{225}{3} = 75°$
$x_2 = \quad = 195°$
$x_3 = \quad = 315°$

$\underline{\underline{z_j = \sqrt[6]{8}\,(\cos x_j + i\sin x_j)}}$

16.

$3\cdot(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi)\cdot 4(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi) = 12\left(\cos(\frac{4}{3}+\frac{5}{6})\pi + i\sin(\frac{4}{3}+\frac{5}{6})\pi\right) =$

$\underline{\underline{= 12\cdot(\cos\frac{13}{6}\pi + i\sin\frac{13}{6}\pi)}} = 12\cdot(\cos\frac{1}{6}\pi + i\sin\frac{1}{6}\pi) = 12\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = \underline{6\sqrt{3} + 6i}$

$3(\cos\frac{5}{3}\pi + i\sin\frac{5}{3}) : 4\cdot(\cos\frac{1}{6}\pi + i\sin\frac{1}{6}\pi) = \frac{3}{4}\left(\cos(\frac{5}{3}-\frac{1}{6})\pi + i\sin(\frac{5}{3}-\frac{1}{6})\pi\right) =$

$= \frac{3}{4}\cdot(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi) = \frac{3}{4}\cdot(0 - i) = \underline{\underline{-\frac{3}{4}i}}$

2b

$\underline{\underline{\Sigma\ 30b}}$