1. Let $\mathbb{A} \in \mathbb{R}^{3\times 3}$ be an invertible matrix and let $\boldsymbol{u}$ and $\boldsymbol{v}$ be arbitrary fixed vectors in $\mathbb{R}^3$ such that $\boldsymbol{v} \bullet \mathbb{A}^{-1}\boldsymbol{u} \neq -1$. Show that

$$\left(\mathbb{A} + \boldsymbol{u} \otimes \boldsymbol{v}\right)^{-1} = \mathbb{A}^{-1} - \frac{1}{1 + \boldsymbol{v} \bullet \mathbb{A}^{-1}\boldsymbol{u}} \left(\mathbb{A}^{-1}\boldsymbol{u}\right) \otimes \left(\mathbb{A}^{-\top}\boldsymbol{v}\right).$$

   The formula is usually referred to as the Sherman–Morrison formula.

2. We already know that

$$\epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km},$$

   and let us further assume that we also know that

$$\epsilon_{ijk}\epsilon_{lmn} = \det \begin{bmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{bmatrix}.$$

   Show that the Levi–Civita symbol satisfies the identities

$$\epsilon_{ijk}\epsilon_{ijn} = 2\delta_{kn},$$
$$\epsilon_{ijk}\delta_{lm} = \epsilon_{jkm}\delta_{il} + \epsilon_{kim}\delta_{jl} + \epsilon_{ijm}\delta_{kl}.$$