1. Ukažte, že složky symetrického gradientu $\mathbb{D}$ vektorového pole $\mathbf{v}$, tedy $\mathbb{D} =_{\text{def}} \frac{1}{2}\left(\nabla \mathbf{v} + (\nabla \mathbf{v})^{\top}\right)$, jsou v cylindrických souřadnicích

$$
\begin{aligned}
x &= r\cos\varphi,\\
y &= r\sin\varphi,\\
z &= z,
\end{aligned}
$$

dány vztahem

$$
\left(\mathbb{D}\right)^{\hat{\imath}}{}_{\hat{\jmath}} = \begin{bmatrix}
\frac{\partial \mathrm{v}^{\hat{r}}}{\partial r} & \frac{1}{2}\left(\frac{1}{r}\frac{\partial \mathrm{v}^{\hat{r}}}{\partial \varphi} + \frac{\partial \mathrm{v}^{\hat{\varphi}}}{\partial r} - \frac{\mathrm{v}^{\hat{\varphi}}}{r}\right) & \frac{1}{2}\left(\frac{\partial \mathrm{v}^{\hat{r}}}{\partial z} + \frac{\partial \mathrm{v}^{\hat{z}}}{\partial r}\right)\\
\frac{1}{2}\left(\frac{1}{r}\frac{\partial \mathrm{v}^{\hat{r}}}{\partial \varphi} + \frac{\partial \mathrm{v}^{\hat{\varphi}}}{\partial r} - \frac{\mathrm{v}^{\hat{\varphi}}}{r}\right) & \frac{1}{r}\frac{\partial \mathrm{v}^{\hat{\varphi}}}{\partial \varphi} + \frac{\mathrm{v}^{\hat{r}}}{r} & \frac{1}{2}\left(\frac{\partial \mathrm{v}^{\hat{\varphi}}}{\partial z} + \frac{1}{r}\frac{\partial \mathrm{v}^{\hat{z}}}{\partial \varphi}\right)\\
\frac{1}{2}\left(\frac{\partial \mathrm{v}^{\hat{r}}}{\partial z} + \frac{\partial \mathrm{v}^{\hat{z}}}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial \mathrm{v}^{\hat{\varphi}}}{\partial z} + \frac{1}{r}\frac{\partial \mathrm{v}^{\hat{z}}}{\partial \varphi}\right) & \frac{\partial \mathrm{v}^{\hat{z}}}{\partial z}
\end{bmatrix}.
$$

Zároveň ukažte, že složky gradientu $\nabla \mathbf{v}$ jsou v cylindrických souřadnicích dány vztahem

$$
\left(\nabla \mathbf{v}\right)^{\hat{\imath}}{}_{\hat{\jmath}} = \begin{bmatrix}
\frac{\partial \mathrm{v}^{\hat{r}}}{\partial r} & \frac{1}{r}\frac{\partial \mathrm{v}^{\hat{r}}}{\partial \varphi} - \frac{\mathrm{v}^{\hat{\varphi}}}{r} & \frac{\partial \mathrm{v}^{\hat{r}}}{\partial z}\\
r\frac{\partial}{\partial r}\left(\frac{\mathrm{v}^{\hat{\varphi}}}{r}\right) + \frac{\mathrm{v}^{\hat{\varphi}}}{r} & \frac{1}{r}\frac{\partial \mathrm{v}^{\hat{\varphi}}}{\partial \varphi} + \frac{\mathrm{v}^{\hat{r}}}{r} & \frac{\partial \mathrm{v}^{\hat{\varphi}}}{\partial z}\\
\frac{\partial \mathrm{v}^{\hat{z}}}{\partial r} & \frac{1}{r}\frac{\partial \mathrm{v}^{\hat{z}}}{\partial \varphi} & \frac{\partial \mathrm{v}^{\hat{z}}}{\partial z}
\end{bmatrix}.
$$

V obou případech se složkami míní "fyzikální" složky příslušných tensorů, aneb složky vůči *normované* bázi.

2. Uvažuje vektory $\mathbf{t}_1$, $\mathbf{t}_2$ v $\mathbb{R}^3$. Ukažte, že platí

$$
\left|\mathbf{t}_1 \times \mathbf{t}_2\right|^2 = \det\begin{bmatrix} \mathbf{t}_1 \bullet \mathbf{t}_1 & \mathbf{t}_1 \bullet \mathbf{t}_2 \\ \mathbf{t}_1 \bullet \mathbf{t}_2 & \mathbf{t}_2 \bullet \mathbf{t}_2 \end{bmatrix}.
$$