1. Let us consider Eulerian description. Some people claim that the balance of mass and balance of momentum read

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{v}) = 0,$$
$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \operatorname{div}(\rho \boldsymbol{v} \otimes \boldsymbol{v}) = \operatorname{div} \mathbb{T} + \rho \boldsymbol{b},$$

whilst the notation is the same as ours (ρ is the density, $\boldsymbol{v}$ Eulerian velocity field, $\mathbb{T}$ Cauchy stress tensor and $\boldsymbol{b}$ is body force). Are these equations equivalent to the equations we have derived at the last lecture? Why?

2. Consider the deformation $\boldsymbol{x} = \boldsymbol{\chi}(\boldsymbol{X}, t)$ given by the following formulae

$$\mathrm{x}_1 = \lambda(t)\mathrm{X}_1,$$
$$\mathrm{x}_2 = [\lambda(t)]^{-\frac{1}{2}}\,\mathrm{X}_2,$$
$$\mathrm{x}_3 = [\lambda(t)]^{-\frac{1}{2}}\,\mathrm{X}_3,$$

where $\lambda(t)$ is a positive function of time, $\lambda(t_0) = 1$. Find explicit formulae for the Lagrangian velocity field $\boldsymbol{V}$, Eulerian velocity field $\boldsymbol{v}$, deformation gradient $\mathbb{F}$, stretch tensor $\mathbb{U}$ and rotation tensor $\mathbb{R}$ from the polar decomposition of $\mathbb{F}$, velocity gradient $\mathbb{L}$, symmetric part of the velocity gradient $\mathbb{D}$, left Cauchy–Green tensor $\mathbb{B}$, right Cauchy–Green tensor $\mathbb{C}$ and Green–Saint-Venant strain $\mathbb{E}$.

Is the deformation isochoric? (Isochoric = preserves volume.)