1. Prove the following lemma. Let $\Omega \subset \mathbb{R}^3$ is a bounded domain with a smooth boundary, and $\boldsymbol{v}$ is a smooth vector field that vanishes on the boundary $\boldsymbol{v}|_{\partial\Omega} = 0$. Then

$$2 \int_{\Omega} \mathbb{D} : \mathbb{D} \, \mathrm{dv} = \int_{\Omega} \nabla \boldsymbol{v} : \nabla \boldsymbol{v} \, \mathrm{dv} + \int_{\Omega} (\operatorname{div} \boldsymbol{v})^2 \, \mathrm{dv},$$

where $\mathbb{D}$ denotes the symmetric part of the gradient of $\boldsymbol{v}$, that is $\mathbb{D} =_{\text{def}} \frac{1}{2} \left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^{\top} \right)$, and $\mathbb{A} : \mathbb{B} =_{\text{def}} \operatorname{Tr} \left(\mathbb{A} \mathbb{B}^{\top} \right)$.