1. Consider an isothermal flow of a compressible inviscid fluid,

$$\mathbb{T} = -p(\rho)\mathbb{I},$$

in the case of potential body forces $\boldsymbol{b} = -\rho \nabla \phi$, where ϕ is the potential.

Prove Bernoulli's theorem. Namely show that the expression

$$\left[\int \frac{\partial \boldsymbol{v}}{\partial t}\right] + \frac{1}{2}\boldsymbol{v}^2 + P + \phi,$$

is constant along *streamlines*. What quantity is denoted by P? What is the meaning of $\int \frac{\partial v}{\partial t}$? The proof should be based on the analysis of the governing equations.