1. Prove the following lemma. Let $\Omega \subset \mathbb{R}^3$ is a bounded domain with a smooth boundary, and $\boldsymbol{v}$ is a smooth vector field that vanishes on the boundary $\boldsymbol{v}|_{\partial\Omega} = 0$. Then
$$2 \int_\Omega \mathbb{D} : \mathbb{D} \, \mathrm{dV} = \int_\Omega \nabla \boldsymbol{v} : \nabla \boldsymbol{v} \, \mathrm{dV} + \int_\Omega (\operatorname{div} \boldsymbol{v})^2 \, \mathrm{dV},$$
where $\mathbb{D}$ denotes the symmetric part of the gradient of $\boldsymbol{v}$, $\mathbb{D} =_{\mathrm{def}} \frac{1}{2}\left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^\top\right)$.