1. Let $\mathbb{A} \in \mathbb{R}^{3\times3}$ be an invertible matrix and let $\boldsymbol{u}$ and $\boldsymbol{v}$ be arbitrary fixed vectors in $\mathbb{R}^3$ such that $\boldsymbol{v} \bullet \mathbb{A}^{-1}\boldsymbol{u} \neq -1$. Show that

$$(\mathbb{A} + \boldsymbol{u} \otimes \boldsymbol{v})^{-1} = \mathbb{A}^{-1} - \frac{1}{1 + \boldsymbol{v} \bullet \mathbb{A}^{-1}\boldsymbol{u}} \left(\mathbb{A}^{-1}\boldsymbol{u}\right) \otimes \left(\mathbb{A}^{-\top}\boldsymbol{v}\right).$$

2. We already know that

$$\epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km},$$

and let us further assume that we also know that

$$\epsilon_{ijk}\epsilon_{lmn} = \det \begin{bmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{bmatrix}.$$

- Show that

$$\epsilon_{ijk}\epsilon_{ijn} = 2\delta_{kn}.$$

- Show that

$$\epsilon_{ijk}\delta_{lm} = \epsilon_{ljk}\delta_{im} + \epsilon_{ilr}\delta_{jm} + \epsilon_{ijl}\delta_{km}.$$

Comment (10th October):

- Symbol $\mathbb{A}^{-\top}$ is defined as $\mathbb{A}^{-\top} = \left(\mathbb{A}^{-1}\right)^{\top}$.
- Obviously, there is a typo in the last formula, the index r should be replaced by k. The formula should read

$$\epsilon_{ijk}\delta_{lm} = \epsilon_{jkm}\delta_{il} + \epsilon_{kim}\delta_{jl} + \epsilon_{ijm}\delta_{kl}.$$