1. Let $\mathbf{v}$ denote the Eulerian velocity field. Show that

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\partial \mathbf{v}}{\partial t} + (\operatorname{rot}\mathbf{v}) \times \mathbf{v} + \nabla\left(\frac{1}{2}\mathbf{v} \bullet \mathbf{v}\right),$$

where $\frac{\mathrm{d}}{\mathrm{d}t}$ in the material time derivative.

2. Let $\mathbb{T}$ denote the Cauchy stress tensor in $\mathbb{R}^2$, $\mathbb{T} = \begin{bmatrix} \mathrm{T}_{\hat{x}\hat{x}} & \mathrm{T}_{\hat{x}\hat{y}} \\ \mathrm{T}_{\hat{y}\hat{x}} & \mathrm{T}_{\hat{y}\hat{y}} \end{bmatrix}$, and let $\mathbf{n}$ denotes the normal to a given surface element, and let $\mathbf{t}$ be any vector such that $\mathbf{t} \bullet \mathbf{n}$. If the surface element is oriented in such a way that $\mathbb{T}\mathbf{n} = \tau\mathbf{n}$, then we say that the surface element with orientation $\mathbf{n}$ experiences a *pure tension/compression* . (The direction of *traction* is parallel to the normal to the surface element.) If the surface element is oriented in such a way that $\mathbb{T}\mathbf{n} \bullet \mathbf{t} \neq \mathbf{0}$, then we say that the surface element with the orientation $\mathbf{n}$ experiences a *shear stress.*

   - Show that it is always possible to find a surface element with normal $\mathbf{n}$ oriented in such a way that the element experiences the *pure tension/compression.*
   - Find the orientation $\mathbf{n}$ of the surface that is subject to the *maximal shear stress.* In other words, what is the value of $\mathbf{n}$ that maximises

     $$\left|\mathbb{T}\mathbf{n} - \left(\left(\mathbb{T}\mathbf{n}\right) \bullet \mathbf{n}\right)\mathbf{n}\right|.$$

     (Try to guess what is the solution before you make the formal computation.)
   - What is the relation between the maximal achievable tension/compression and the maximal achievable shear stress? (Try to guess what is the solution before you make the formal computation.)

   It would be nice to get the answers in a coordinate free form, say by referring only to the eigenvalues and eigenvectors of the Cauchy stress tensor $\mathbb{T}$.