1. Consider $\boldsymbol{\chi}$ given by the following formulae

$$
\begin{aligned}
r &= f(R),\\
\varphi &= \Phi,\\
z &= Z.
\end{aligned}
$$

This means that the deformation $\boldsymbol{\chi}$ is given as a function that takes coordinates $[R, \Phi, Z]$ in the reference configuration—with respect to the cylindrical coordinate system—and returns position of that point in terms or polar coordinates in the current configuration, see Figure 1.

Show that the deformation gradient is given by the formula

$$
\mathbb{F} = \frac{\mathrm{d}f}{\mathrm{d}R} \mathbf{e}_{\hat{r}} \otimes \mathbf{E}_{\hat{R}} + \frac{f}{R} \mathbf{e}_{\hat{\varphi}} \otimes \mathbf{E}_{\hat{\Phi}} + \mathbf{e}_{\hat{z}} \otimes \mathbf{E}_{\hat{Z}}.
$$

that is

$$
\mathbb{F} = \begin{bmatrix} \frac{\mathrm{d}f}{\mathrm{d}R} & 0 & 0 \\ 0 & \frac{f}{R} & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

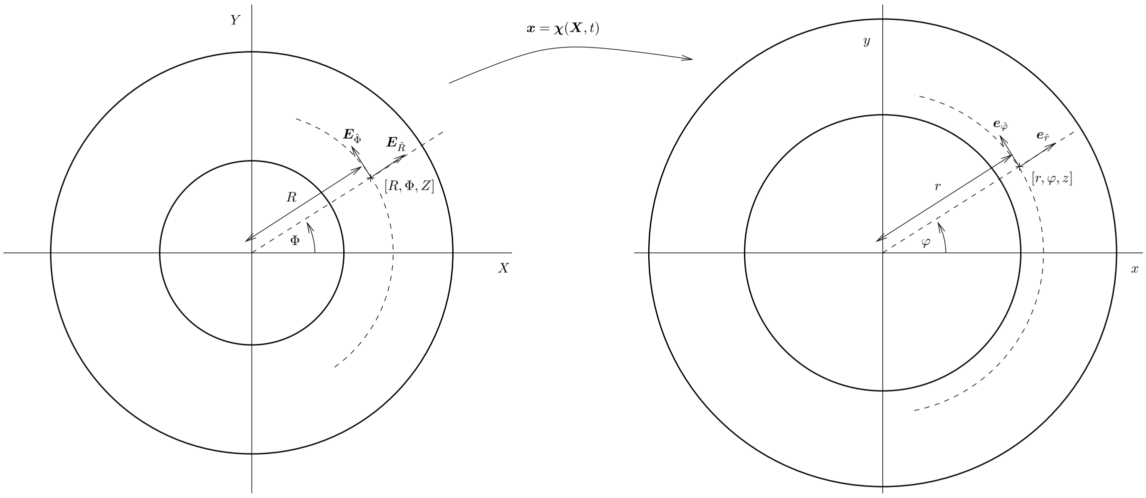

Figure 1: Problem geometry.

2. Recall that the Euler–Almansi strain tensor is defined as $\mathbb{e}(\mathbf{x}, t)|_{\mathbf{x}=\boldsymbol{\chi}(\mathbf{X},t)} =_{\text{def}} \frac{1}{2}\left(\mathbb{I} - \mathbb{F}^{-\top}(\mathbf{X}, t)\mathbb{F}^{-1}(\mathbf{X}, t)\right)$. Show that the material time derivative of Euler–Almansi strain tensor is given by the formula

$$
\frac{\mathrm{d}\mathbb{e}}{\mathrm{d}t} = \mathbb{D} - \mathbb{L}^\top \mathbb{e} - \mathbb{e}\mathbb{L}.
$$