NMMO 401 Continuum mechanics

Winter 2015/2016

1. Find the surface integral

$$I = \int_{S} \mathbf{v} \bullet \mathrm{d}\mathbf{s},\tag{1}$$

where the vector field $\mathbf{v}(x, y, z)$ is given by the formula

$$\mathbf{v} = \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix},\tag{2}$$

and the surface S is the boundary of the volume $V = \{\mathbf{x} \in \mathbb{R}^3 | x^2 + y^2 \le z^2, z \in [0, h]\}$. (The orientation of the surface is in the direction of the outward normal to the volume V.) After finishing the calculations find by direct computation the volume integral

$$J = \int_{V} \operatorname{div} \mathbf{v} \, \mathrm{dv}. \tag{3}$$

Verify, that I = J.