Higher integrability for doubly nonlinear parabolic equations

Juha Kinnunen

We discuss a local higher integrability result for the spatial gradient of weak solutions to doubly nonlinear parabolic equations of the type

$$\big(|u|^{p-2}u\big)_t - \operatorname{div}\big(|Du|^{p-2}Du\big) = 0$$

in the range

$$\max\left\{\frac{2n}{n+2}, 1\right\} < p < \frac{2n}{(n-2)_+},$$

where $n \in \mathbb{N}$ is the spatial dimension. In [1] we show that a gradient of a nonnegative weak solution to a doubly nonlinear equation belongs locally to a slightly higher Sobolev space than assumed a priori with a reverse Hölder type estimate. The range may seem unexpected, but the lower bound also appears in the higher integrability for the parabolic p-Laplace equation [6], while the upper bound is the same as for the porous medium equation in the fast diffusion range [5] and [3].

The equation is homogeneous in the sense that solutions are invariant under multiplication by constants, but constants cannot be added without destroying the property of being a solution. The key ingredient in the proof of our main result is an appropriate intrinsic geometry that depends on the the solution as well as its spatial gradient and thus allows us to rebalance the mismatch between the function and its gradient in the equation. Variants of this techinque have been successfully used in proving the correspoding results for the parabolic p-Laplace equation [6] and for the porous medium equation [4], [5], [2] and [3]. Related results and open questions are also discussed.

References

[1] V. Bögelein, F. Duzaar, J. Kinnunen and C. Scheven, Higher integrability for doubly nonlinear parabolic systems (submitted).

[2] V. Bögelein, F. Duzaar, R. Korte and C. Scheven, The higher integrability of weak solutions of porous medium systems, *Adv. Nonlinear Anal.* **8**(1): 1004–1034, 2019.

[3] V. Bögelein, F. Duzaar and C. Scheven, Higher integrability for the singular porous medium system (submitted).

[4] U. Gianazza and S. Schwarzacher, Self-improving property of degenerate parabolic equations of porous medium-type, *Amer. J. Math.* **141**: 399–446, 2019.

[5] U. Gianazza and S. Schwarzacher, Self-improving property of the fast diffusion equation (submitted)

[6] J. Kinnunen and J.L. Lewis, Higher integrability for parabolic systems of p-Laplacian type, *Duke Math. J.*, **102**(2): 253–271, 2000.