

A. Vyšetřete bodovou, stejnoměrnou a lokálně stejnoměrnou konvergenci:

$$1. f_n(x) = \frac{n^2 x^2}{1 + n^2 x^2}, \quad 2. f_n(x) = \frac{x^n}{1 + x^n}, \quad x > -1,$$

$$3. f_n(x) = x^n - x^{n+1}, \quad x \geq 1, \quad 4. f_n(x) = \sqrt[n]{1 + x^n}, \quad x \geq 0,$$

$$5. f_n(x) = \exp(-(x - n)^2), \quad 6. f_n(x) = x^n \exp(-nx),$$

$$7. f_n(x) = x^n \exp(-nx), \quad x > 0 \quad 8. f_n(x) = \left(1 + \frac{x}{n}\right)^n, \quad x \geq 0,$$

$$9. f_n(x) = \frac{nx}{1 + n + x}, \quad x \geq 0, \quad 10. f_n(x) = \sin(\pi x^n), \quad |x| \leq 1,$$

$$11. f_n(x) = \frac{\arctan(nx)}{nx}, \quad x > 0, \quad 12. f_n(x) = \frac{x}{n} \ln\left(\frac{x}{n}\right), \quad x > 0,$$

$$13. f_n(x) = n \left(\sin\left(x + \frac{1}{n}\right) - \sin x \right), \quad 14. f_n(x) = \sqrt[2n]{x^n + \ln|x|}, \quad x > 0$$

$$15. f_n(x) = n \ln\left(1 + \frac{x}{n}\right), \quad x > -1, \quad 16. f_n(x) = \frac{1 + x^{n+1}}{1 + x^n}, \quad x > -1,$$

$$17. f_n(x) = \sqrt[n]{x + n}, \quad x > -1, \quad 18. f_n(x) = \frac{\ln(nx)}{nx}, \quad x > 0.$$

Vyšetřete (lokálně) (absolutně) stejnoměrnou konvergenci řad $\sum_k f_k(x)$, kde

$$1. f_k(x) = k^p z^k, \quad z \in C, \quad k \in Z, \quad 2. f_k(x) = \sin\left(\frac{x}{2^k}\right),$$

$$*3. f_k(x) = \frac{kx^{k-1}}{1+x^k} \sin\left(\frac{x}{k}\right), \quad x \in (0, 1), \quad 4. f_k(x) = \frac{(-1)^k}{k^{1+\frac{1}{2}}} \frac{x^k}{1+x^k},$$

$$5. f_k(x) = \frac{(-1)^k}{kx^2+1}, \quad 6. f_k(x) = (x^2+k^2)^{-1/2} \cos \frac{2k\pi}{3},$$

$$7. f_k(x) = (-1)^k \frac{\sqrt{k}}{k+100} \frac{\cosh(kx) + \sinh(kx)}{\cosh(kx)}, \quad 8. f_k(x) = (-1)^k (1-x)x^k,$$

$$9. f_k(x) = \sin^2 x \cos^k x, \quad 10. f_k(x) = \sin\left(x^{ke} \exp(-kx)\right), \quad x \geq 0,$$

$$11. f_k(x) = \exp(-kx) \sin(kx^2), \quad x \geq 0, \quad 12. f_k(x) = \frac{kx}{k^2+x^2} \operatorname{arctg}\left(\frac{x}{k}\right),$$

$$13. f_k(x) = \operatorname{arccotg}\left(k^x + k^{1/x}\right), \quad x \geq 0, \quad 14. f_k(x) = \frac{\sin x \sin(kx)}{\sqrt{x+k}}, \quad x \geq 0,$$

$$15. f_k(x) = \operatorname{arctg}\left(\frac{2x}{x^2+k^3}\right), \quad 16. f_k(x) = x^\alpha \exp(-kx), \quad \alpha > 0, \quad x \geq 0,$$

$$17. f_k(x) = \exp(-k^2x), \quad 18. f_k(x) = \sin\left(\pi\sqrt{\alpha^2+k^2}\right) \sqrt[k]{\frac{x^2}{1+x^2}}.$$