Ordinary Differential Equations

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Problem Set 6

6.1. Using the polar coordinates, determine if the origin is a stable or asymptotically stable equilibrium of

$$x' = -2y + ax\sqrt{x^2+y^2}$$
$$y' = 2x + ay\sqrt{x^2+y^2}$$

depending on $a \in \mathbb{R}$.

6.2. (a) Decide about stability of solutions (not only stationary points) to

$$x' = -(x^2+y^2)y$$
$$y' = (x^2+y^2)x.$$

Note: Recall that a solution $\bar{x}$ is called *(Lyapunov) stable* if for each $\varepsilon > 0$ and $t_0 \in \mathbb{R}$ there exists $\delta = \delta(\varepsilon, t_0) > 0$ such that if $x(t)$ is a solution and $|x(t_0) - \bar{x}(t_0)| < \delta$ then $|x(t_0) - \bar{x}(t_0)| < \varepsilon$ for all $t \geq t_0$.

(b) If the solutions are not stable, can you define your own notion of stability here so that the previously unstable solutions become stable in this new sense of yours?

Note: Unfortunately for you, this notion already has a name – *orbital stability.*

6.3. Draw the phase portrait of the following equation given in polar coordinates:

$$r' = r(1-r)$$
$$\varphi' = \sin^2(\varphi/2).$$

Are the stationary solutions stable or locally attractive? Can you find a connection to Vinogradov's example?

6.4. Sketch the phase portrait of the linearized system in the neighbourhood of equilibria:

$$x' = 2x + y^2 - 1$$
$$y' = \sin(x) - y^2 - 1.$$

What phase portraits do you expect from the original system around these points?

6.5. **Food for thought:** Three gods A, B and C are called, in no particular order, True, False and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter every time he speaks. Your task is to determine the identities of A, B and C by asking three yes-no questions; each question must be put to exactly one god. A single god may be asked more than one question and questions are permitted to depend on the answers to earlier questions. The gods understand Czech, but will answer all questions in their own language, in which the words for *yes* and *no* are *brrr* and *grrr*, in some order. You do not know which word means which.