1. Dokažte:

   (a) $1 + 2 + \dots + n = \frac{1}{2}n(n+1)$

   (b) $1^2 + 2^2 + \dots n^2 = \frac{1}{6}n(n+1)(2n+1)$

   (c) $1^3 + 2^3 + \dots n^3 = (1 + 2 + \dots n)^2$

   (d) $1 + q + q^2 + \dots q^n = \frac{1-q^{n+1}}{1-q}, \qquad q \neq 1$

   (e) $\sum_{k=2}^{n} \frac{1}{k^2} < \frac{n-1}{n+1}$

   (f) $\sum_{k=1}^{n} \frac{1}{k} \leq 2\sqrt{n}$

   (g) $(1+x)^n \geq 1 + nx, \quad x \geq -1$

   (h) $\sum_{k=1}^{n} \frac{1}{\sqrt{k}} > \sqrt{n}, \quad n \geq 2$

2. Řešte nerovnice:

   (a) $(x-3)(x+1)(3x+6) > 0$

   (b) $\frac{2x+1}{x-3} > 4$

   (c) $|x+1| + |x-2| \leq 4$

   (d) $\frac{x-1}{x-2} > \frac{x-3}{x-4}$

   (e) $\frac{1}{x} + \frac{1}{1-x} < 2$

   (f) $|x^2 - 4x + 3| \leq |x^2 - 4|$

   (g) $\frac{|x+1|}{x-1} \geq x$

3. Dokažte rovnosti:

   (a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

   (b) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

   (c) $C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B)$

   (d) $C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$

   (e) $A \setminus (A \setminus B) = A \cap B$

   (f) $A \setminus (B \cup C) = (A \setminus B) \setminus C$

4. Zkoumejte vztah následujících množin:

   (a) $f(A \cup B)$ a $f(A) \cup f(B)$

   (b) $f(A \cap B)$ a $f(A) \cap f(B)$

   (c) $f(f^{-1}(A))$ a A

   (d) $f^{-1}(f(A))$ a A

   (e) $f^{-1}(A \cup B)$ a $f^{-1}(A) \cup f^{-1}(B)$

   (f) $f^{-1}(A \cap B)$ a $f^{-1}(A) \cap f^{-1}(B)$

5. Určete maximum, minimum, infimum a supremum množin:

(a) $M := \{1/(x-1) :\ x \geq 0\}$

(b) $M := \{\frac{n}{n+1} :\ n \in N\}$

(c) $M := \{\sin \frac{n\pi}{4} :\ n \in N\}$

(d) $M := \left\{(-1)^n \cdot \frac{n^2+1}{n^2-1} :\ n \in N\right\}$

(e) Ukažte, že $\sup\{-a :\ a \in A\} = -\inf A$.

(f) Ukažte, že $\sup\{a + b :\ a \in A,\ b \in B\} = \sup A + \sup B$.

6. Dokažte následující vlastnosti absolutní hodnoty:

(a) $|ab| = |a||b|$

(b) $|a/b| = |a|/|b| \quad (b \neq 0)$

(c) $-|a| \leq a \leq |a|$

(d) $|a| = a \operatorname{sgn} a$

(e) $|a \pm b| \leq |a| + |b|$

(f) $|a \pm b| \geq ||a| - |b||$

DŮLEŽITÉ LIMITY

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x \to 0} \frac{\exp x - 1}{x} = 1 \qquad \lim_{x \to \infty} \frac{\ln x}{x} = 0$$

$$\lim_{x \to \infty} \frac{x}{\exp x} = 0$$