(1) $f(x) = \dfrac{x^2-2x+1}{x^2-3x+2} = \dfrac{x-1}{x-2} \to 0; \quad x \to 1$

(2) $f(x) = \dfrac{1}{x^2-2x} - \dfrac{x}{x^2-4} = -\dfrac{x+1}{x^2+2x} \to -\dfrac{3}{8}, \quad x \to 2$

(3) $f(x) = \dfrac{(1+x)(1+2x)(1+3x)-1}{x} = 6+11x+6x^2 \to 6, \quad x \to 0.$

(4) $f(x) = \dfrac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}} = \dfrac{(x+1)^{20}}{(x+4)^{10}} \to \left(\dfrac{3}{2}\right)^{10}; \quad x \to 2$

(5) $f(x) = \dfrac{x^{m-1}+x^{m-2}+\dots+x+1}{x^{n-1}+x^{n-2}+\dots+x+1} \to \dfrac{m}{n}; \quad x \to 1$

dle vzorce: $a^n - b^n = (a-b)\cdot\left(a^{n-1}+a^{n-2}b+\dots+ab^{n-2}+b^{n-1}\right)$

(6) $f(x) = \dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4} = \dfrac{1}{\sqrt[4]{x}+2} \to \dfrac{1}{4}, \quad x \to 16$

(7) $f(x) = \dfrac{\sqrt{1-2x-x^2}-(1+x)}{x} = \dfrac{-4}{\sqrt{1-2x-x^2}+(1+x)} \to -2, \quad x \to 0$

dle vzorce $\sqrt{a} - \sqrt{b} = \dfrac{a-b}{\sqrt{a}+\sqrt{b}}$;

(8) $f(x) = \dfrac{(1+x)^n-1}{x} = n + x\underbrace{p(x)}_{\text{polynom}} \to n, \quad x \to 0$

dle bin. věty $(n \geq 2)$

$$(1+x)^n = 1 + nx + \underbrace{\sum_{k=2}^{n} \binom{n}{k} x^k}_{x^2 p(x)}$$

9* $f(x) = \dfrac{\sqrt[n]{1+x} - 1}{x} = \dfrac{1}{\left(\sqrt[n]{1+x}\right)^{n-1} + \left(\sqrt[n]{1+x}\right)^{n-2} + \dots + 1} \to \dfrac{1}{n}$

$x \to 0$

dle vzorce $\sqrt[n]{a} - \sqrt[n]{b} = \dfrac{a-b}{\left(\sqrt[n]{a}\right)^{n-1} + \left(\sqrt[n]{a}\right)^{n-2}\sqrt[n]{b} + \dots + \left(\sqrt[n]{b}\right)^{n-1}}$

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10 $f(x) = \dfrac{\sqrt[3]{1+x^3} - 1}{x^2} = \dfrac{x}{\left(\sqrt[3]{1+x^3}\right)^2 + \sqrt[3]{1+x^3} + 1} \to 0, \ x \to 0$

dle vzorce $\sqrt[3]{a} - \sqrt[3]{b} = \dfrac{a-b}{a^2+ab+b^2}$

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11 $f(x) = \dfrac{\sqrt{1+x} - \sqrt[3]{1+x}}{x} = \dfrac{\sqrt{1+x} - 1}{x} - \dfrac{\sqrt[3]{1+x} - 1}{x} \to \dfrac{1}{2} - \dfrac{1}{3}$

dle příkl. 9*

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12 $f(x) = \dfrac{\sqrt{x} - 1}{x-1} = \dfrac{1}{\sqrt{x}+1} \to \dfrac{1}{2} \ ; \ x \to 1$

(13) $f(x)=\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sin x^2}=\dfrac{2x^2}{\sin x^2}\cdot\dfrac{1}{\sqrt{1+x^2}+\sqrt{1-x^2}}\to 2\cdot 1\cdot\dfrac{1}{1+1}=\underline{\underline{1}}.$

$\dfrac{y}{\sin y}=\dfrac{1}{\frac{\sin y}{y}}\to\dfrac{1}{1}=1;\ y\to 0$ (známá limita)

$x^2\to 0,\ x\to 0;$ leč $x^2\neq 0$ na $P(0,\delta)$.

V. 2.6, (b) $\Rightarrow\ \dfrac{x^2}{\sin x^2}\to 1;\ x\to 0.$

$1\pm x^2\to 1;\ x\to 0$

$g(y)=\sqrt{y}$ .. spojitá v $y_0=1$.

V. 2.6; (a) $\Rightarrow\ \sqrt{1\pm x^2}\to\sqrt{1}=1;\ x\to 0$

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(14) $f(x)=\dfrac{1}{x}\left[\ln(1+x)+\ln(1-x)\right]=(-x)\dfrac{\ln(1-x^2)}{-x^2}\to -0\cdot 1=0$
$x\to 0$

$$\left.\begin{aligned}&\frac{\ln(1+y)}{y}\to 1;\ y\to 0\\&-x^2\to 0;\ x\to 0\\&\quad\text{leč }\neq 0\text{ na }P(0,\delta)\end{aligned}\right\}\ \text{V. 2.6 (b)}\Rightarrow\ \frac{\ln(1-x^2)}{-x^2}\to 1,\quad x\to 0$$

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(15) $f(x)=\dfrac{e^{x\cdot\ln a}-1}{x}=\ln a\cdot\dfrac{e^{x\cdot\ln a}-1}{x\cdot\ln a}\to\ln a;\ x\to 0.$

$$\left.\begin{aligned}&\frac{e^y-1}{y}\to 1;\ y\to 0\\&x\cdot\ln a\to 0,\ x\to 0\\&\quad x\cdot\ln a\neq 0;\ x\in P(0,\delta)\end{aligned}\right\}\ \text{V. 2.6. (b)}\Rightarrow\ \frac{e^{x\cdot\ln a}-1}{x\cdot\ln a}\to 1,\ x\to 0.$$

(16) $f(x) = \dfrac{x^b - a^b}{x-a} = a^b \dfrac{\left(\frac{x}{a}\right)^b - 1}{x-a} = a^b \dfrac{e^{b \cdot \ln\frac{x}{a}} - 1}{x-a}$

$b \neq 0$.

$$= a^b \cdot \underbrace{\frac{e^{b\cdot \ln\frac{x}{a}} - 1}{b \cdot \ln\frac{x}{a}}}_{(1)} \cdot \underbrace{\frac{\ln\frac{x}{a}}{\frac{x}{a} - 1}}_{(2)} \cdot \frac{b}{a} \to b \cdot a^{b-1};$$

$\dfrac{e^y - 1}{y} \to 1$; $y \to 0$

$b \cdot \ln\frac{x}{a} \to b \cdot \ln 1 = 0$; $x \to a$

$b \cdot \ln \underbrace{\frac{x}{a}}_{\neq 1} \neq 0$; $x \in P(a)$

$\overset{V.\,2.6\,(b)}{\Longrightarrow}$ $(1) \to 1$

$\lim\limits_{y \to 1} \dfrac{\ln y}{y-1} = 1$

$\frac{x}{a} \to 1$; $x \to a$

$\neq 1$ na $P(a)$

$\Longrightarrow$ $(2) \to 1$.

(17) $f(x) = \dfrac{b^x - b^a}{x-a} = b^a \dfrac{b^{x-a} - 1}{x-a} = b^a \dfrac{e^{(x-a)\ln b} - 1}{(x-a)\ln b} \cdot \ln b$

$b \neq 1$

$\to b^a \cdot \ln b$,

nebot

$\dfrac{e^y - 1}{y} \to 1$; $y \to 0$

$(x-a)\ln b \to 0$; $x \to a$ .. Věta 2.6. (b)

$\neq 0$ na $P(a)$

(18) $f(x) = \frac{\ln \cos x}{x^2} = \underbrace{\frac{\ln \cos x}{\cos x - 1}}_{(i)} \cdot \underbrace{\frac{\cos x - 1}{x^2}}_{(ii)} \to 1 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$

$x \to 0$

$$\left.\begin{array}{l} \frac{\ln y}{y-1} \to 1; \ y \to 1 \\ \cos x \to 1; \ x \to 0 \\ \cos x \neq 1 \text{ na } P(0) \end{array}\right\} \overset{V.2.6\,(b)}{\Longrightarrow} (i) \to 1$$

$$\frac{\cos x - 1}{x^2} = \frac{\cos x - 1}{x^2} \cdot \frac{\cos x + 1}{\cos x + 1} = \frac{\cos^2 x - 1}{x^2} \cdot \frac{1}{\cos x + 1}$$

$$= -\frac{\sin^2 x}{x^2} \cdot \frac{1}{\cos x + 1} = -\underbrace{\left(\frac{\sin x}{x}\right)^2}_{(iii)} \cdot \underbrace{\frac{1}{\cos x + 1}}_{(iv)} \to -\frac{1}{2}, x \to 0.$$

nebo: $\left.\begin{array}{l} \frac{\sin x}{x} \to 1; \ x \to 0 \\ g(y) = y^2 \text{ spojitá v } y_0 = 1 \end{array}\right\} \overset{V.2.6\,(a)}{\Longrightarrow} (iii) \to 1$

$\cos x \to 1; \ x \to 0$ (spojitost $\cos x$)

$\Rightarrow \frac{1}{\cos x + 1} \to \frac{1}{1+1} = \frac{1}{2}$ (aritmetika limit), $x \to 0$.

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(19) $f(x) = \frac{\sin \pi x}{x - 1} = -\frac{\sin \pi(x-1)}{\pi(x-1)} \cdot \pi \to -\pi; \ x \to 1$

{trik: $\sin(y \pm \pi) = -\sin y$}

$\frac{\sin y}{y} \to 1; \ y \to 0$

$\pi(x-1) \to 0; \ x \to \pi$

$\neq 0; \ x \in P(1)$

(20) $\lim\limits_{x \to 0} \underbrace{\dfrac{\arcsin x}{x}}_{f(t)} = 1$;

$f(t) = g(h(t))$; $\quad g(y) = \dfrac{y}{\sin y} \to 1; \; y \to 0$

$h(x) = \arcsin x \to 0 \; ; \; x \to 0$

$h(t) \neq 0$ na $P(0)$. $\qquad$ V.2.6, (b).

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(21) $f(x) = \dfrac{\cos x}{x - \frac{\pi}{2}} = \dfrac{\sin\left(\frac{\pi}{2} - x\right)}{x - \frac{\pi}{2}} \to -1$

{ [illegible]: $\cos x = \sin\left(\frac{\pi}{2} - x\right)$

$$\left.\begin{array}{l} g(y) = \dfrac{\sin y}{y} \to 1; \quad y \to 0 \\ f(t) = x - \frac{\pi}{2} \to 0; \; x \to \frac{\pi}{2} \\ \quad \neq 0 \text{ na } P\left(\frac{\pi}{2}\right) \end{array}\right\} \text{.. řešení dále V.2.6 (b).}$$