A. Vyšetřete konvergenci integrálů:

1. $\int_0^\infty \frac{\sqrt[3]{x}+3}{\sqrt{x}+2}\,dx$ 2. $\int_0^1 \frac{\sin(\pi x)}{x^{9/8}\ln x}\,dx$ 3. $\int_0^\infty \frac{\ln(x+1)}{x^{101/100}}\,dx$

4. $\int_0^\infty \frac{\sin^2 x}{x^a}\,dx$ 5. $\int_0^\infty \sin^3\left(\frac{1}{\sqrt{x}}\right)dx$ 6. $\int_0^\pi \frac{dx}{\sqrt{\sin x}}$

7. $\int_0^\infty x\exp(-x)\sin x\,dx$ $*$ 8. $\int_0^\infty \frac{x}{1+x^5\sin^2 x}\,dx$ 9. $\int_0^{1/2} \frac{dx}{x\ln^b x}$

10. $\int_0^\infty \frac{dx}{\exp x-\cos x}$ 11. $\int_0^\infty \frac{\operatorname{arctg} x}{x^a}\,dx$ $*$ 12. $\int_0^\infty \frac{p(x)}{q(x)}\,dx$

B. Záměnou limity a integrálu vypočtěte:

1. $\lim_{n\to\infty}\int_0^\infty \exp(-x^n)\,dx$ 2. $\lim_{n\to\infty}\int_0^{10} \frac{nx}{1+n^2x^2}\,dx$

3. $\lim_{n\to\infty}\int_0^\infty \frac{dx}{1+x+x^2n^{-2}}$ 4. $\lim_{n\to\infty}\int_0^\infty \frac{\operatorname{arctg}(nx)}{1+x^3}\,dx$

5. $\lim_{n\to\infty}\int_1^\infty \frac{dx}{\ln x+\ln n}$ 6. $\lim_{n\to\infty}\int_0^\infty \frac{x^n}{1+x^{2n}}\,dx$

7. $\lim_{n\to\infty}\int_0^\infty \frac{\ln(x+n)}{n}\exp(-x)\cos x\,dx$ 8. $\lim_{n\to\infty}\int_0^{23} \frac{\exp(x^3)}{1+n\sqrt{x}}\,dx$

C. Rozviňte v řadu:

1. $\int_0^1 \ln\left(\frac{1+x}{1-x}\right)dx$ 2. $\int_0^\infty \exp(-x)\cos\sqrt{x}\,dx$ 3. $\int_0^1 \ln(x)\ln(1-x)\,dx$

4. $\int_0^1 \frac{x^p\ln x}{1+x^2}\,dx$ 5. $\int_0^\infty \frac{\sin x}{1+\exp x}\,dx$ 6. $\int_0^\infty \frac{x}{\exp x-1}\,dx$