NMFM401 – Mathematics of Non-Life Insurance 1

8. Aggregate Loss Models II

A Exercises

1. On a given day, a physician provides medical care to N_A adults and N_C children. N_A and N_C have Poisson distributions with parameters 3 and 2, respectively. The distribution of length of care per patient are as follows.

	Adult	Child
1 hour	0.4	0.9
2 hours	0.6	0.1

N_A, N_C, and the lengths of care for all individuals are independent. The physician charges 200 per hour of patient care. Determine the probability that the office income on a given day is less or equal to 800.

2. You are given two independent compound Poisson random variables S_1 and S_2, where $f_j(x)$, $j = 1, 2$, are the two single-claim size distributions. You are given $\lambda_1 = \lambda_2 = 1$, $f_1(1) = 1$; and $f_2(1) = f_2(2) = 0.5$. Let $F_X(x)$ be the single-claim size distribution function associated with the compound distribution $S = S_1 + S_2$. Calculate $F_X^{*4}(6)$.

3. For a compound Poisson distribution with positive integer claim amounts, the probability function follows:

$$f_S(x) = \frac{1}{x}[0.16 f_S(x-1) + k f_S(x-2) + 0.72 f_S(x-3)], \quad x = 1, 2, \ldots.$$

The expected value of aggregate claims is 1.68. Determine the expected number of claims.

4. Individual members of an insured group have independent claims. The claim distribution has the statistics given in the following table.

	Mean	Variance
Males	2	4
Females	4	10

The premium for a group with future claims S is the mean of S plus 2 times the standard deviation of S. If the genders of the members of a group of m members are not known, the number of males are assumed to have a binomial distribution with parameters m and $q = 0.4$. A is the premium for a group of 100 for which the genders of the members are not known. B is the premium for a group of 40 males and 60 females. Determine A/B.

B Homework

1. A group of policyholder's aggregate claims, S, has a compound Poisson distribution with $\lambda = 1$ and all claim amounts equal to 2. The insurer pays the group the following dividend:

$$D = \begin{cases} 6 - S, & S < 6; \\ 0, & S \geq 6. \end{cases}$$

Determine $\mathbb{E}D$.

2. S has a compound Poisson claims distribution with the following:

(a) Individual claim amounts equal to 1,2, or 3.
(b) $\mathbb{E}S = 56$.
(c) $\mathbb{V}ar\, S = 126$.
(d) $\lambda = 29$.

Determine the expected number of claims of size 2.

3. For a portfolio of policies you are given the following:

(a) The number of claims has a Poisson distribution.
(b) Claim amounts can be 1,2, or 3.
(c) A stop-loss reinsurance has net premiums for various deductibles as follows:

Deductible	Net premium
4	0.20
5	0.10
6	0.04
7	0.02

Determine the probability that aggregate claims will be either 5 or 6.

4. For group disability income insurance, the expected number of disabilities per year is 1 per 100 lives covered. The continuous (survival) function for the length of a disability in days, Y, is $\mathbb{P}[Y > y] = 1 - y/10$, $y = 0, 1, \ldots, 10$. The benefit is 20 per day following a 5-day waiting period. Using a compound Poisson distribution, determine the variance of aggregate claims for a group of 1,500 independent lives.

5. Based on the individual risk model with independent claims, the cdf of aggregate claims for a portfolio of life insurance policies is as follows.

x	$F_S(x)$
0	0.40
100	0.58
200	0.64
300	0.69
400	0.70
500	0.78
600	0.96
700	1.00

One policy with face amount 100 and probability of claim 0.20 is increased in face amount to 200. Determine the probability that aggregate claims for the revised portfolio will not exceed 500.

(Version: October 1, 2014)

Source

[1] Klugman, Stuart A., Panjer, Harry H., and Willmont, Gordon E. *Loss Models: From Data to Decisions.* John Wiley & Sons, Inc., New York, NY, 4th Edition, 2012.