NMFM401 – Mathematics of Non-Life Insurance 1

7. Aggregate Loss Models I

A Exercises

1. When an individual is admitted to the hospital, the hospital charges have the following characteristics:

(a)

Charges	Mean	Standard deviation
Room	1,000	500
Other	500	300

(b) The covariance between an individual's room charges and other charges is 100,000.

An insurer issues a policy that reimburses 100% for Room Charges and 80% for Other Charges. The number of hospital admissions has a Poisson distribution with parameter 4. Determine the mean and standard deviation of the insurer's payout for the policy.

2. X_1, X_2, and X_3 are mutually independent loss random variables with probability functions as given in the table below.

x	$f_1(x)$	$f_2(x)$	$f_3(x)$
0	0.90	0.50	0.25
1	0.10	0.30	0.25
2	0.00	0.20	0.25
3	0.00	0.00	0.25

Determine the pf of $S = X_1 + X_2 + X_3$.

3. A weighted average of two Poisson distributions

$$\mathbb{P}[N = k] = w\frac{e^{-\lambda_1}\lambda_1^k}{k!} + (1-w)\frac{e^{-\lambda_2}\lambda_2^k}{k!}$$

has been used to threat drivers as either “good” or “bad”.

(a) Find the pgf $P_N(z)$ of the number of losses in terms of the two pgfs $P_1(z)$ and $P_2(z)$ of the number of losses of the two types of drivers.

(b) Let $f_X(x)$ denote a severity distribution defined on the non-negative integers. How can relation

$$f_S(x) = \frac{\lambda}{x}\sum_{y=1}^{x\wedge r} y f_X(y) f_S(x-y), \quad x = 1, 2, \ldots$$

be used to compute the distribution of aggregate claims for the entire group?

(c) Can this be extended to other frequency distributions?

4. For a compound Poisson distribution, $\lambda = 6$ and individual losses have pf $f_X(1) = f_X(2) = f_X(4) = 1/3$. Some of the pf values for the aggregate distribution S are:

x	$f_S(x)$
3	0.0132
4	0.0215
5	0.0271
6	$f_S(6)$
7	0.0410

Determine $f_S(6)$.

B Homework

1. Automobile drivers can be divided into three homogeneous classes. The number of claims for each driver follows a Poisson distribution with parameter λ. Determine the variance of the number of claims for a randomly selected driver.

Class	Proportion of population	λ
1	0.25	5
2	0.25	3
3	0.50	2

2. X_1, X_2, and X_3 are mutually independent loss random variables with probability functions as given in the table below.

x	$f_1(x)$	$f_2(x)$	$f_3(x)$
0	p	0.6	0.25
1	$1-p$	0.2	0.25
2	0	0.1	0.25
3	0	0.1	0.25

If $S = X_1 + X_2 + X_3$ and $f_S(5) = 0.06$, determine p.

3. A compound Poisson aggregate loss model has 5 expected claims per year. The severity distribution is defined on positive multiples of 1,000. Given that $f_S(1) = e^{-5}$ and $f_S(2) = \frac{5}{2}e^{-5}$, determine $f_X(2)$.

4. Aggregate claims are compound Poisson with $\lambda = 2$, $f_X(1) = 1/4$, and $f_X(2) = 3/4$. For a premium of 6 an insurer covers aggregate claims and agrees to pay a dividend (a refund of premium) equal to the excess, if any, of 75% of the premium over 100% of the claims. Determine the excess of premium over expected claims and dividends.

(Version: November 19, 2014)

Source

[1] Klugman, Stuart A., Panjer, Harry H., and Willmont, Gordon E. *Loss Models: From Data to Decisions.* John Wiley & Sons, Inc., New York, NY, 4th Edition, 2012.