NMFM401 – Mathematics of Non-Life Insurance 1

6. Frequency III – Models for the Number of Payments

A Exercises

1. For the data from the following table determine the MoM estimates of the parameters of the Poisson-Poisson distribution where the secondary distribution is the ordinary (not-zero truncated) Poisson distribution. Perform the chi-squared GoF test using the model.

No. of accidents	Observed frequency
0	103,704
1	14,075
2	1,766
3	255
4	45
5	6
6	2
7+	0

2. Show that for any pgf, $P^{(k)}(1) = \mathbb{E}[N(N-1)\cdots(N-k+1)]$, provided the expectation exists. Here $P^{(k)}(z)$ indicates the kth derivative. Use this result to conform that the first three central moments of the compound Poisson distribution are

$$\mu_j = \lambda m'_j, \quad j = 1, 2, 3,$$

where m'_j is the jth raw moment of the secondary distribution.

3. Show that the pgf for the inverse Gaussian distribution

$$f(x;\mu,\theta) = \left(\frac{\theta}{2\pi x^3}\right)^{1/2} \exp\left\{-\frac{\theta(x-\mu)^2}{2x\mu^2}\right\}, \quad x > 0,\ \mu > 0,\ \theta > 0$$

is

$$P(z) = \exp\left\{-\frac{\theta}{\mu}\left[\sqrt{1 - 2\frac{\mu^2}{\theta}\log z} - 1\right]\right\}.$$

4. Individual losses have a Pareto distribution

$$f(x;\alpha,\theta) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}, \quad x \geq 0,\ \alpha > 0,\ \theta > 0$$

with $\alpha = 2$ and $\theta = 1{,}000$. With a deductible of 500 the frequency distribution for the number of payments is Poisson-inverse Gaussian with $\lambda = 3$ and $\beta = 2$. If the deductible is raised to 1,000, determine the distribution for the number of payments. Also, determine the pdf of the severity distribution (per payment) when the new deductible is in place.

B Homework

1. In Exercise Hw 4 from the previous assignment, the best model from among the members of the $(a, b, 0)$ and $(a, b, 1)$ classes was selected for the four data sets. Fit the Poisson-Poisson, Polya-Aeppli, Poisson-inverse Gaussian, and Poisson-ETNB (generalized Poisson-Pascal) distribution to these data and use the chi-squared GoF test as well as the LRT to determine if any of these distributions should replace the one selected in Exercise Hw 4 from the previous assignment. Is the current best model acceptable.

2. Verify that for the Poisson-binomial distribution, the first three central moments are

$$\begin{aligned} \mu &= \lambda m q, \\ \sigma^2 &= \mu[1 + (m-1)q], \\ \mu_3 &= 3\sigma^2 - 2\mu + \frac{m-2}{m-1}\frac{(\sigma^2-\mu)^2}{\mu}. \end{aligned}$$

3. Show that the negative binomial-Poisson compound distribution is the same as a mixed Poisson distribution with a negative binomial mixing distribution.

4. Losses have a Pareto distribution with $\alpha = 2$ and $\theta = 1{,}000$. The frequency distribution for a deductible of 500 is zero-truncated logarithmic with $\beta = 4$. Determine a model for the number of payments when the deductible is reduced to 0.

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Source

[1] Klugman, Stuart A., Panjer, Harry H., and Willmont, Gordon E. *Loss Models: From Data to Decisions.* John Wiley & Sons, Inc., New York, NY, 4th Edition, 2012.