NMFM401 – Mathematics of Non-Life Insurance 1

5. Frequency II – Models for the Number of Payments

A Exercises

1. Assume that the binomial parameter m from the binomial model is known. Consider the MLE of q.

(a) Show that the MLE is unbiased.

(b) Determine the variance of the MLE.

(c) Show that the asymptotic variance provided by the Fisher's information is the same as that developed in the previous part.

(d) Determine a formula for a confidence interval in two different ways: based on replacing q by $\widehat{q}$ in the variance term; and without using previously mentioned replacement.

2. The following table gives the number of medical claims per reported automobile accident.

No. of medical claims	No. of accidents	No. of medical claims	No. of accidents
0	529	5	87
1	146	6	41
2	169	7	25
3	137	8+	0
4	99		

(a) Construct a plot of ratios kn_k/n_{k-1} against k, where n_k is the number of policies (or accidents) with k claims. Does it appear that a member of the $(a, b, 0)$ class will provide a good model? If so, which one?

(b) Determine the MLEs of the parameters for each member of the $(a, b, 0)$ class.

(c) Based on the chi-squared GoF test and the likelihood ratio test, which member of the $(a, b, 0)$ class provides the best fit? Is this method acceptable?

3. Determine the iterative formula for the scoring method for the zero-modified Poisson distribution derived from the zero-truncated Poisson distribution

$$\mathbb{P}[N = k] = \frac{\lambda^k}{k!(\exp\{\lambda\} - 1)}, \quad \lambda > 0,\ k = 1, 2, \ldots$$

and perform one iteration for the following data using starting values $p_0^M = 0.879337$ and $\lambda = 0.178267$.

Accidents	Observed	Accidents	Observed
0	370,412	4	28
1	46,545	5	3
2	3,935	6+	0
3	317		

Confirm that these starting values are the MLEs and obtain the estimated covariance matrix.

4. Repeat the previous exercise for the zero-modified logarithmic distribution derived from the (zero-truncated) logarithmic distribution

$$\mathbb{P}[N = k] = \frac{\beta^k}{k(1+\beta)^k \log(1+\beta)}, \quad \beta > 0,\ k = 1, 2, \ldots$$

with starting values $p_0^M = 0.879337$ and $\beta = 0.189011$. Conduct the chi-square GoF. Is this model superior to the zero-modified geometric distribution?

B Homework

1. A portfolio of 10,000 risks produced the following numbers of claims. For parts (a)–(c) assume that the value of m is known to be 4.

No. of claims	No. of policies	No. of claims	No. of policies
0	9,048	3	2
1	905	4+	0
2	45		

(a) Determine the MLE of q of the binomial model.

(b) Construct the chi-squared GoF test, using the MLE.

(c) Construct the 95% confidence intervals for q using the methods developed in part (d) of Exercise Ex 1.

(d) Determine the MLEs of m and q by constructing a likelihood profile.

(e) Conduct the likelihood ratio test to determine if a binomial model is more appropriate than a Poisson model.

(f) Of Poisson, negative binomial, geometric, and binomial, which model fits the data best? Explain.

2. Use the following data. For parts (a)–(c) assume that the value of m is known to be 7.

No. of claims	No. of policies	No. of claims	No. of policies
0	861	4	1
1	121	5	0
2	13	6	1
3	3	7+	0

(a) Determine the MLE of q of the binomial model.

(b) Construct the chi-squared GoF test, using the MLE.

(c) Construct the 95% confidence intervals for q using the methods developed in part (d) of Exercise Ex 1.

(d) Determine the MLEs of m and q by constructing a likelihood profile.

(e) Conduct the likelihood ratio test to determine if a binomial model is more appropriate than a Poisson model.

(f) Of Poisson, negative binomial, geometric, and binomial, which model fits the data best? Explain.

3. For each of the data sets in Exercises Hw 1 and Hw 2 construct a plot similar to that in Exercise Ex 2. For each graph, determine the most appropriate model from the $(a, b, 0)$ class. Compare your response to your answer to part (f) of Exercises Hw 1 and Hw 2.

4. For the four data sets introduced in earlier Exercises Hw 1, Hw 2, Ex 2, and Ex 3, you have determined the best model from among members of the $(a, b, 0)$ class. For each data set determine the MLEs of the zero-modified Poisson, geometric, logarithmic, and negative binomial distribution, where the last mentioned distribution is derived from zero-truncated negative binomial distribution

$$\mathbb{P}[N = k] = \frac{r(r+1)\dots(r+k-1)}{k![(1+\beta)^r - 1]} \left(\frac{\beta}{1+\beta}\right)^k, \quad \beta > 0,\ r > -1,\ k = 1, 2, \dots.$$

Use the chi-squared GoF test and likelihood ratio tests to determine the best of the eight models considered and state whether or not the selected model is acceptable.

(Version: October 28, 2013)

Source

[1] Klugman, Stuart A., Panjer, Harry H., and Willmont, Gordon E. *Loss Models: From Data to Decisions.* John Wiley & Sons, Inc., New York, NY, 4th Edition, 2012.