NMFM401 – Mathematics of Non-Life Insurance 1

2. Severity II – Models for the Amount of a Single Payment

A Exercises

1. The amount of a single claim has a Pareto distribution with pdf

$$f(x;\alpha,\theta)=\frac{\alpha\theta^{\alpha}}{x^{\alpha+1}},\quad x>\theta,\ \alpha>0,\ \theta>0$$

and $\alpha = 2$ and $\theta = 2{,}000$. Determine the loss elimination ratio for a deductible of 500.

2. The distribution of losses for claims in 2012 has a discrete distribution with

$$\mathbb{P}[X = 1{,}000k] = 1/6,\ k = 1,\dots,6.$$

The insurance contract calls for a deductible of 1,500 on each loss. Inflation of 5% impacts all claims uniformly from 2012 to 2013. The deductible remains at 1,500 in 2013. Determine the percentage increase in expected payments per loss from 2012 to 2013.

3. A random sample of auto glass claims has yielded the following five observed claims amounts: 100, 125, 200, 250, and 300. Determine the value of the empirical mean excess loss function at $x = 250$.

4. It has been conjectured that losses have a Pareto distribution with parameters $\alpha = 2$; $\theta = 1{,}000$ and pdf

$$f(x;\alpha,\theta)=\frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}},\quad x>0,\ \alpha>0,\ \theta>0.$$

A random sample of size 10 produced 3 losses in the range $0 - 250$, 2 in the range $250 - 500$, 3 in the range $500 - 1{,}000$, and 2 above 1,000. Perform the chi-square goodness-of-fit test with a significance level of 0.10. [Do not require that expected counts be at least 5.]

5. Given sample values of 0.1, 0.4, 0.8, 0.8, and 0.9 you wish to test the goodness of fit of the distribution with pdf $f(x) = (1 + 2x)/2$, $0 \le x \le 1$. Determine the Kolmogorov-Smirnov test statistic and conduct the test at 5% significance level.

B Homework

1. The distribution of losses for claims in 2012 is lognormal with pdf

$$f(x;\mu,\sigma)=\frac{\exp\{-z^2/2\}}{x\sqrt{2\pi\sigma^2}},\quad z=\frac{\log x-\mu}{\sqrt{\sigma^2}},\quad x>0,\ \mu\in\mathbb{R},\ \sigma^2>0$$

and $\mu = 10$ and $\sigma^2 = 5$. From 2012 to 2013, an inflation rate of 10% impacts all claims uniformly. The insurer has purchased reinsurance that pays the excess over 2,000,000 on any claim. Determine the insurer's expected payment per loss for 2013.

2. Claim amounts in 2012 have the normal distribution with $\mu = 1{,}000$ and $\sigma^2 = 10{,}000$. Inflation of 5% impacted all claims uniformly from 2012 to 2013. Determine the distribution for claim amounts in 2013.

3. For 2012, loss sizes follow a uniform distribution on $[0; 2{,}500]$. Inflation of 3% impacts all losses uniformly from 2012 to 2013. In 2013 a deductible of 100 is applied to all losses. Determine the loss elimination ratio for 2013.

4. It has been conjectured that general liability payments (300,000 limit) have a lognormal distribution with ML estimates $\widehat{\mu} = 9.29376$ and $\sqrt{\widehat{\sigma}^2} = 1.62713$. Amounts paid from policies are listed in the table below:

Pyments	Number	Average
0 – 2,500	41	1,389
2,500 – 7,500	48	4,661
7,500 – 12,500	24	9,991
12,500 – 17,500	18	15,482
17,500 – 22,500	15	20,232
22,500 – 32,500	14	26,616
32,500 – 47,500	16	40,278
47,500 – 67,500	12	56,414
67,500 – 87,500	6	74,985
87,500 – 125,000	11	106,851
125,000 – 225,000	5	184,735
225,000 – 300,000	4	264,025
at 300,00	3	300,000

Perform the chi-square goodness-of-fit test to determine if the lognormal model is appropriate.

5. In Homework 1.B.6. a gamma distribution with ML estimates $\widehat{\alpha} = 6.341$ and $\widehat{\theta} = 599.3$ was fitted to the data. Use the Kolmogorov-Smirnov test to determine if the gamma model is reasonable.

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Source

[1] Klugman, Stuart A., Panjer, Harry H., and Willmont, Gordon E. *Loss Models: From Data to Decisions.* John Wiley & Sons, Inc., New York, NY, 4th Edition, 2012.