NMFM401 – Mathematics of Non-Life Insurance 1

1. Severity I – Models for the Amount of a Single Payment

A Exercises

1. The cdf of a random variable is $F(x) = 1 - x^{-2}$, $x \geq 1$. Determine the mean, median, and the mode of this random variable.

2. The severities of individual claims have the Pareto distribution with pdf

$$f(x;\alpha,\theta) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}, \quad x \geq 0,\ \alpha > 0,\ \theta > 0$$

and parameters $\alpha = 8/3$ and $\theta = 8{,}000$. Use the CLT to approximate the probability that the sum of 100 independent claims will exceed 600,000.

3. A sample of 1,000 health insurance contracts on adults produced a sample mean of 1,300 for the annual benefits paid with a standard deviation of 400. It is expected that 2,500 contracts will be issued next year. Use the CLT to estimate the probability that benefit payments will be more than 101% of the expected amount.

4. A population of losses has the Pareto distribution with $\theta = 6{,}000$ and $\alpha > 0$ unknown. Simulation of the results from maximum likelihood estimation based on samples of size 10 has indicated $\mathbb{E}\widehat{\alpha} = 2.2$ and $MSE(\widehat{\alpha}) = 1$. Determine $\mathbb{V}ar\,\widehat{\alpha}$ if it is known that $\alpha = 2$.

5. Let $X_1, \dots, X_n$ be a random sample from a population with cdf $F(x) = x^p 1_{(0,1)}(x)$.

(a) Determine the MLE of p.

(b) Determine the MoM estimate of p.

(c) Determine the asymptotic variance of the maximum likelihood estimator of p.

(d) Use your answer to obtain a general formula for a 95% confidence interval for p.

(e) Determine the maximum likelihood estimator of $\mathbb{E}X$ and obtain its asymptotic variance and a formula for a 95% confidence interval.

6. A random sample of claims has been drawn from a loglogistic distribution with pdf

$$f(x;\gamma,\theta) = \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2}, \quad x \geq 0,\ \gamma > 0,\ \theta > 0.$$

In the sample, 80% of the claims exceed 100 and 20% exceed 400. Estimate the loglogistic parameters by percentile matching.

B Homework

1. There have been 30 claims recorded in a random sampling of claims. There were 2 claims for 2,000, 6 for 4,000, 12 for 6,000, and 10 for 8,000. Determine the empirical skewness coefficient.

2. The severity distribution of individual claims has pdf $f(x) = 2.5x^{-3.5}$, $x \geq 1$. Determine the coefficient of variation.

3. A sample of size 500 has been taken from a continuous distribution. A confidence interval for the median is formed by using the order statistics $X_{(240)}$ a $X_{(260)}$. Use the normal approximation to determine the level of confidence. [Hint: Approximate the binomial distribution by the normal one.]

4. Six independent observations, $X_1, \ldots, X_6$, were obtained. The variance was estimated as $\widehat{\sigma}^2 = \sum_{j=1}^{6}(X_j - \bar{X})^2/6$ with $\bar{X} = \sum_{j=1}^{6} X_j/6$. The true population variance is $\sigma^2 = 2$. Determine the bias of $\widehat{\sigma}^2$.

5. Let $X_1, \ldots, X_n$ be a random sample from a population with pdf $f(x) = \theta^{-1}\exp\{-x/\theta\}$, $x \geq 0, \theta > 0$.

(a) Determine the MLE of θ.

(b) Determine the asymptotic variance of the maximum likelihood estimator of θ.

(c) Use your answer to obtain a general formula for a 95% confidence interval for θ.

(d) Determine the maximum likelihood estimator of $\mathbb{V}ar\, X$ and obtain its asymptotic variance and a formula for a 95% confidence interval.

6. A random sample of 10 claims obtained from a gamma distribution with pdf

$$f(x; \alpha, \theta) = \frac{(x/\theta)^\alpha \exp\{-x/\theta\}}{x\Gamma(\alpha)}, \quad x > 0,\ \alpha > 0,\ \theta > 0$$

is given: 1,500; 6,000; 3,500; 3,800; 1,800; 5,500; 4,800; 4,200; 3,900; 3,000.

(a) Estimate α and θ by the MoM.

(b) Suppose it is known that $\alpha = 12$. Determine the MLE of θ.

(c) Determine the MLE of α and θ.

7. The random variable X has pdf $f(x; \alpha, \lambda) = \alpha\lambda^\alpha(\lambda + x)^{-\alpha-1}$, $x \geq 0, \alpha > 0, \lambda > 0$. It is known that $\lambda = 1{,}000$. You are given the following five observations: 43, 145, 233, 396, 775.

(a) Determine the MoM estimate of α.

(b) Determine the MLE of α.

(c) Estimate the variance of the MLE.

(d) Use the estimated variance from above to construct a 95% confidence interval for $\mathbb{E}\min\{X, 500\}$.

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Source

[1] Klugman, Stuart A., Panjer, Harry H., and Willmont, Gordon E. *Loss Models: From Data to Decisions.* John Wiley & Sons, Inc., New York, NY, 4th Edition, 2012.