

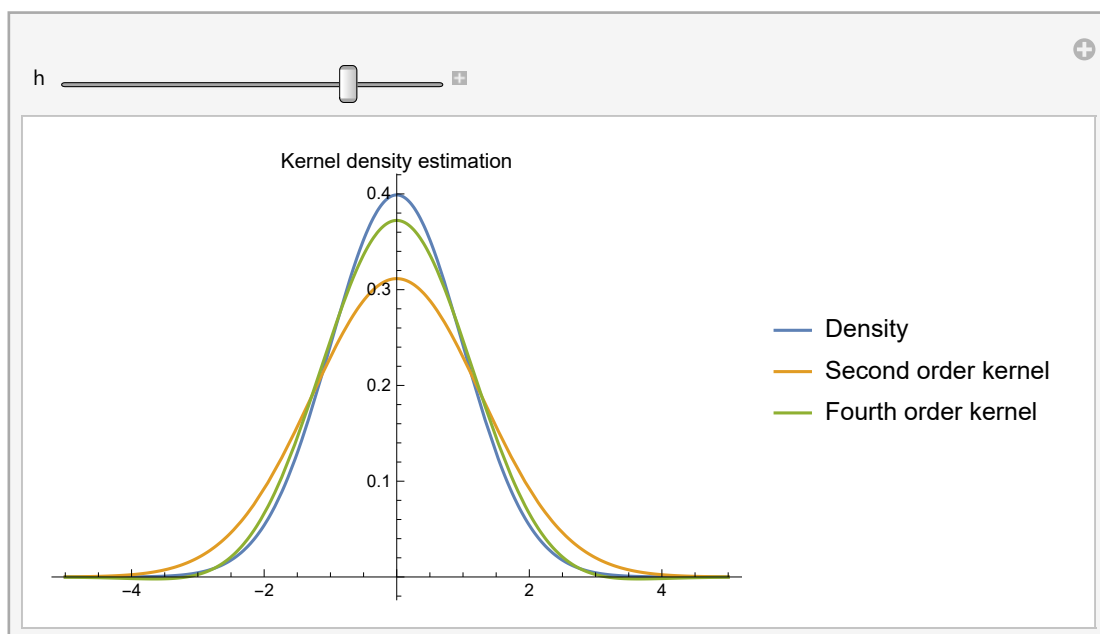
NMST434 : Exercise session XII

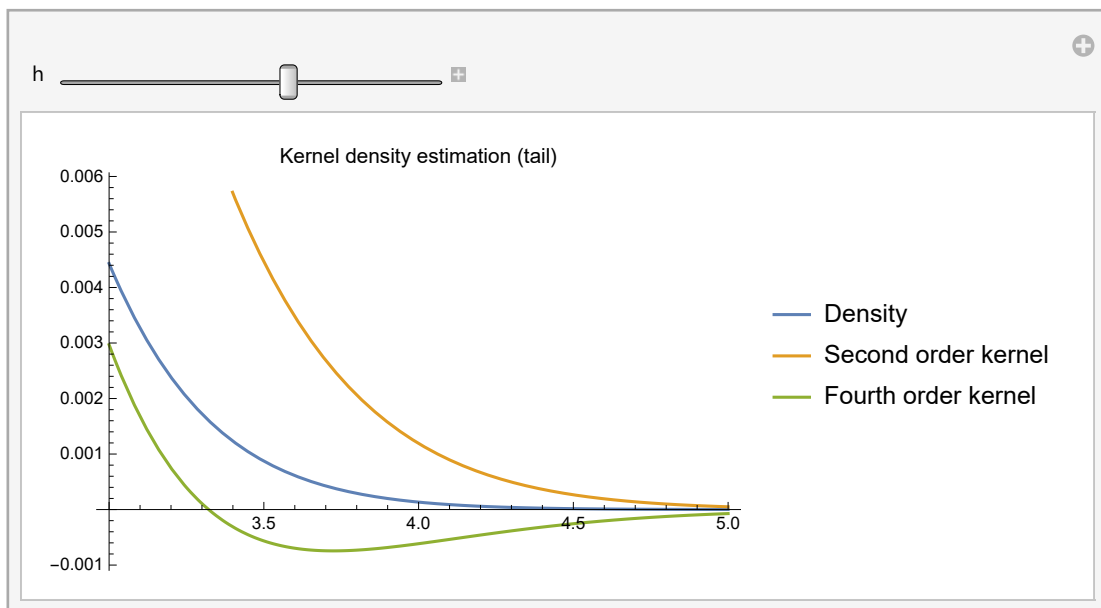
May 22, 2018

Kernel density estimation: bias and higher order kernels

Gaussian density

```
f[x_] = PDF[NormalDistribution[], x]; (* True density *)
K[t_] = PDF[NormalDistribution[], t]; (* Basic kernel *)
Kb[t_] =  $\frac{1}{2} (3 - t^2) \frac{1}{\sqrt{2\pi}} \text{Exp}[-t^2/2]$ ; (* Higher order kernel *)
fh[x_, h_] =  $\frac{1}{h} \text{Integrate}[K[\frac{x-y}{h}] f[y], \{y, -\infty, \infty\}]$ ;
fhb[x_, h_] =  $\frac{1}{h} \text{Integrate}[Kb[\frac{x-y}{h}] f[y], \{y, -\infty, \infty\}]$ ;
Manipulate[Plot[{f[x], fh[x, h], fhb[x, h]}, {x, -5, 5},
  PlotLegends -> {"Density", "Second order kernel", "Fourth order kernel"},
  PlotLabel -> "Kernel density estimation"], {h, 0.1, 1}]
Manipulate[Plot[{f[x], fh[x, h], fhb[x, h]}, {x, 3, 5},
  PlotLegends -> {"Density", "Second order kernel", "Fourth order kernel"},
  PlotLabel -> "Kernel density estimation (tail)"], {h, 0.1, 1}]
```





Exponential density

```
fb[x_] = PDF[GammaDistribution[1, 1], x]; (* True density *)
```

```
K[t_] = PDF[NormalDistribution[], t]; (* Basic kernel *)
```

```
fhc[x_, h_] =  $\frac{1}{h}$  Integrate[K[ $\frac{x-y}{h}$ ] fb[y], {y, -∞, ∞}];
```

```
Manipulate[
```

```
Plot[{fb[x], fhc[x, h]}, {x, -5, 5}, PlotLegends → {"Density", "Second order kernel"},  
PlotLabel → "Kernel density estimation", PlotRange → All], {h, 0.1, 1}]
```

