

MATHEMATICS I Exam, February 5, 2013

1. Compute the limit of the sequence or prove it does not exist:

$$\lim_{n \rightarrow \infty} \frac{\left(\sqrt{n^2 + 2n + 3} - \sqrt{n^2 + 3n + 4}\right) \cdot \arcsin\left(\frac{\pi}{n+\pi}\right)}{\log \frac{1}{n} - \log \frac{1}{n+1}}.$$

13 points

2. Compute the limit of the function or prove it does not exist:

$$\lim_{x \rightarrow 0+} \frac{\operatorname{tg}(x + \pi) + \operatorname{arctg}(x + 1) + \frac{\log(\cos \sqrt{x})}{x}}{\sqrt[3]{x}}.$$

12 points

3. Find domain of the given function, determine the set of all points in which it is continuous (continuous from the left/from the right). Compute its derivative (oneside derivatives) at each point where it exists.

$$f(x) = \begin{cases} 0 & \text{for } x \in (-\infty, 0), \\ \max\{x^4, 2 - x^2\} & \text{for } x \in \langle 0, +\infty \rangle. \end{cases}$$

Hint: investigate first, where (on which intervals) the function equals x^4 and where $2 - x^2$.

17 points

4. Investigate properties of the function and draw its graph:

$$f(x) = \operatorname{arctg} \frac{2x}{1 + x^2}.$$

16 points

Write detailed arguments for each nontrivial step and verify assumptions of theorems you use.